Adverse Selection, Liquidity Shortage, and Government Liquidity Facilities

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(Preliminary draft)

Abstract

I propose a general equilibrium model with asymmetric information in asset quality. Agents trade illiquid private assets subject to adverse selection and liquid government bonds. The equilibrium features shortage in liquidity and suboptimal investment, the severity of which endogenously responds to aggregate shocks. Government liquidity facilities that issue liquid government bonds to purchase illiquid private assets can alleviate the adverse selection and relax financing constraints. I find large quantitative effects of liquidity facilities on credit market conditions, aggregate investment and output in the Great Recession.

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1 Introduction

The global financial crisis started with an abrupt surge in uncertainty about the value of assets on financial institutions' balance sheets. As collateral asset values plummeted, many banks and financial institutions faced surging funding liquidity risk. The role of adverse selection in asset markets during the crisis has been highlighted (e.g., Tirole, 2012; Morris and Shin, 2012). Some studies have built asymmetric information and endogenous asset illiquidity into quantitative macroeconomic models (Eisfeldt and Rampini, 2006; Kurlat, 2013; Bigio, 2015) and explored their roles in resource allocation, investment, and other macroeconomic activities.

As the Federal Funds Rate reached its zero lower bound, the Federal Reserve resorted to a range of unconventional monetary policy measures. Shortly following the Lehman failure in September 2008, the Federal Reserve initiated the first round of large-scale asset purchases (also known as QE1), where it purchased agency mortgage-backed securities (AMBS), agency debt, and long-term government bonds, with AMBS ultimately accounting for the bulk of the purchases. In addition, the Federal Reserve established various liquidity facilities that effectively provided liquid government assets in exchange for illiquid private assets, such as the Term Auction Facility, the Primary Dealer Credit Facility, and the Term Securities Lending Facility.

The effects of large-scale asset purchases and liquidity facilities have been extensively studied in models with various types of financial frictions (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011, 2013; Del Negro et al., 2017, among many others). However, the implications of adverse selection for the effectiveness of these unconventional policies have not received much attention. By affecting the demand of troubled assets and the liquidity composition of financial institutions' balance sheets, these policies influence the severity of adverse selection in the financial market, the liquidity and price of financial assets and credit supply of financial institutions. As a result, the presence of adverse selection can affect the effectiveness of these policies.

This paper builds a tractable model that captures adverse selection in the asset market and use it to evaluate the effects of unconventional monetary policies. In this model, entrepreneurs hold the economy's physical capital (private assets) and government bonds. Every period, some entrepreneurs receive investment opportunities that allow them to produce capital goods using consumption goods. To finance the production of capital goods, investing entrepreneurs can sell their holdings of private assets and government bonds to non-investing entrepreneurs. Private assets have heterogeneous qualities, which are known to the seller but not to the buyers. Government bonds on the other hand do not suffer from information problem.

In the pooling equilibrium of the private asset market where assets of different qualities are sold at the same price, the private asset is illiquid compared to government bonds, as reflected in a liquidity premium in the government bond price. Aggregate investment is suboptimal, as adverse selection prevents investing entrepreneurs from selling enough private assets to fund investment. The illiquidity of private assets endogenously rises following a negative aggregate TFP shock or a shock that increases the dispersion of asset quality. Following such shocks, the price and trading volume of private assets plummet, causing contractions in real activities.

I use the model to study the effect of the Fed's asset purchases and liquidity facilities. These policies are modeled as government's purchases of illiquid private assets by issuing liquid government bonds. The government's demand for the private assets increases asset price and alleviates the degree of adverse selection. The provision of liquid government bonds increases the liquidity on entrepreneurs' balance sheets and the demand for illiquid private assets in the future. In these ways, asset purchases and liquidity facilities endogenously attenuate adverse selections in private illiquid assets.

By calibrating the model to the 2008 crisis, I show that these unconventional policies have sizable quantitative effects. Absent these measures, aggregate investment would have dropped by an additional 2.3% on impact with even larger medium-run effect, and the output losses would have doubled. The key mechanism for the success is through the entrepreneurs' liquidity. Without these policies, entrepreneurs' sales of illiquid assets would have dropped by much larger.

As shown in prior studies, asset purchases and liquidity facilities can increase the asset price and alleviate financing constraints in the absence of adverse selection. In order to highlight the interaction between the unconventional policies and adverse selection, I show that the adverse selection introduces a wedge between the cost of capital of investing and non-investing entrepreneurs. In the spirit of Kurlat (2013), the model is formally equivalent to an economy with symmetric information but a tax on financial transactions and subsidies on holding capital. The rates of tax and subsidies depend on the quality of capital sold, and therefore they respond to aggregate shocks and liquidity facilities. I show that in this equivalence economy, asset purchases and liquidity facilities would be less effective if these taxes and subsidies were held constant (that is, if liquidity facilities did not alleviate the adverse selection problem).

Beside the aforementioned literature, this paper is related to recent works in the finance literature on asymmetric information in the secondary asset market, and its effect on capital allocation and liquidity hoarding (e.g., Fuchs et al., 2016; Malherbe, 2014). House and Masatlioglu (2015) examines the effect of liquidity policies in a two-period model. In comparison, this paper builds a dynamic general equilibrium model and studies the quantitative effects of large-scale asset purchases and liquidity facilities. Malherbe (2014) finds that more liquidity can worsen the adverse selection problem in asset market as firms hoard cash and reduce demand for these assets. In my model, liquidity (public bond) provision can also cause a flight to liquidity and a reduction in demand for private assets, but quantitatively this effect is outweighed by the central bank's purchases of private assets that increases their demand.

This paper also builds on the literature on idiosyncratic risks, liquidity and investment (Aiyagari, 1994; Woodford, 1990). Del Negro et al. (2017) and Kiyotaki and Moore (2019) study the effect of monetary policy when agents face resalability constraints of private assets and public assets serve as a source of liquidity. In this paper, adverse selection provides a natural micro-foundation to resalability constraints.

Lastly, this paper connects with the literature on the interplay between uncertain shocks and financial frictions. For example, Christiano et al. (2014) studies a DSGE model with dispersion shocks to project returns. They find that a greater dispersion leads to lower investment and that dispersion shocks are an important driving force of business cycle. Unlike this paper, the type of financial friction they consider is costly state verification.

2 The model economy

Consider a discrete-time and infinite-horizon economy. The economy is populated by four types of agents: workers, final good producers, entrepreneurs and the government.

Workers. Consider a unit measure of identical workers who supply labor in a competitive labor market. It is assumed that these workers do not have access to the financial market, and they simply spend their wage income on consumption in each period.[1] Therefore, in each period, workers maximize their per-period utility by choosing consumption (C^w) and labor supply (H):

$$\max_{C^w, H} \quad C^w - \frac{H^{1+\epsilon}}{1+\epsilon},$$

subject to the budget constraint

$$C^w = wH - T, \tag{1}$$

where T is a lump-sum tax. The optimal labor supply decision is simply

$$H^\epsilon = w.$$

Final good producers. The representative final good producer carries out production using aggregate capital K and and labor H. The production function has constant-returns-to-scale:

$$Y = AK^\alpha H^{1-\alpha}.$$

A is the total factor productivity (TFP). The final good producer rents capital from entrepreneurs, hires workers in competitive markets and maximizes the gross profits

$$Y - wH - rK.$$

The optimal labor demand and capital demand conditions are

$$(1-\alpha)\left(\frac{K}{H}\right)^\alpha = w, \tag{2}$$

and

$$\alpha\left(\frac{K}{H}\right)^{\alpha-1} = r. \tag{3}$$

[1]Similar to Kiyotaki and Moore (2019), it can be demonstrated that in the steady-state neighborhood where adverse selection matters and bonds and capital are priced at a premium, workers would opt not to hold any government bonds or capital.

Entrepreneurs. There are a unit measure of entrepreneurs who have a logarithm utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(c_t).$$

Entrepreneurs are presented with an opportunity to make investment (produce capital goods). In each period, with probability π an entrepreneur can produce i units of capital from i units of consumption goods, while with probability $1 - \pi$ an entrepreneur does not have any technology to produce new capital. Entrepreneurs who possess the investment opportunity are referred to as investors, while those who do not are known as savers. This investment opportunity shock is independent and identically distributed across entrepreneurs and through time. In addition, the realization of the shock remains private information to the entrepreneurs.

Following production in each period, the existing capital is made up of a continuous range of pieces, which depreciate at different rates. Each piece of capital is identified by λ, the remaining fraction after depreciation (one minus depreciation rate). In other words, λ is the efficiency units that will remain from a piece of capital. I refer to λ as the quality of capital. I assume that λ follows the log-normal distribution and denote the PDF by $f_\phi(\lambda)$. ϕ is the dispersion (standard deviation) of capital quality. I make two assumptions for tractability reasons. First, $f_\phi(\lambda)$ is the same across entrepreneurs. Second, the quality of each piece of capital is independent across time.

Capital market and information friction. The sequence of events within a period t is depicted in Figure 1. Once aggregate shocks have been realized, and production has taken place, the quality of each piece of capital is revealed to its owner, the entrepreneur. Additionally, the entrepreneur becomes aware of the realized investment opportunity shock. Following this, a capital market opens up, enabling entrepreneurs to buy and sell capital goods from one another. It is worth noting that pieces of capital can be sold separately. I use an index function $\iota(\lambda) : R^+ \to \{0, 1\}$ to denote the decision whether to sell the piece of capital indexed by λ. That is, an entrepreneur with existing capital k sells $k \int_0^\infty \iota(\lambda) f_\phi(\lambda) d\lambda$ units of capital in the capital market.

When a given piece is sold, λ cannot be observed by the buyer. That is, only the seller knows the amount of efficiency units that will remain after depreciation from the particular piece he sells.[2] Assuming anonymity of market participants, I focus on the pooling equilibrium where units of

[2] I do not allow selling a representative portfolio of capital, which would have no adverse selection.

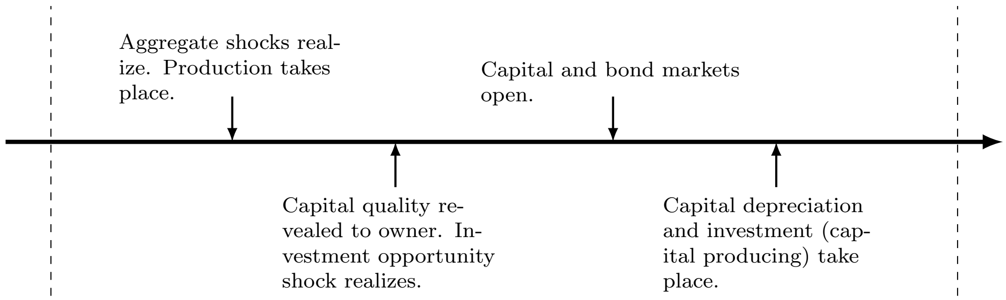

Figure 1: Timeline of activities within period t.

capital of different qualities are sold at the same price q. I use λ^M to denote the average quality of capital sold on the market. The buyers observe the quantity of capital being bought but not its quality. I assume that buyers purchase capital from sufficiently diversified sellers, and their purchase features market quality λ^M. Buyers have rational expectations about λ^M. As the capital quality is independent across time, the asymmetric information only lasts for one period.

Beside capital goods, entrepreneurs can also hold real government bonds in their portfolio. Each unit of bond pays one unit of consumption goods in the next period and trades at a discount price q^B in the current period. The bond market opens at the same time as the capital market.

An investor with existing capital k and bond b solves the following problem.

$$
\begin{aligned}
V^i(k, b; X) = & \max_{c^i, d^i, k^{i\prime}, b^{i\prime}, \iota^i(\lambda), i} \log(c^i) + \beta \mathbb{E}\left[\pi V^i(k^{i\prime}, b^{i\prime}; X') + (1-\pi) V^s(k^{i\prime}, b^{i\prime}; X') | X\right], \\
\text{s.t.} \quad & c^i + i + q d^i + q^B b^{i\prime} = b + rk + qk \int_0^\infty \iota^i(\lambda) f_\phi(\lambda) d\lambda, \\
& k^{i\prime} = k \int_0^\infty \left[1 - \iota^i(\lambda)\right] \lambda f_\phi(\lambda) d\lambda + i + \lambda^M d^i, \\
& d^i \geq 0, \\
& b^{i\prime} \geq 0,
\end{aligned}
$$

where d^i is the purchase of capital from the capital market and λ^M is the average quality of capital purchased. I use X to denote the aggregate state variables and use superscript i to denote the value function or policy function of an investor.

An investor spends her income from bond holding b, rental income of capital rk and revenue of selling capital $qk \int_0^\infty \iota^i(\lambda) f_\phi(\lambda) d\lambda$ on consumption c, investment i, capital purchase qd^i and bond purchase $q^B b^{i\prime}$. Her future-period efficiency units of capital k' is composed of three parts: the amount of existing capital that is unsold and undepreciated $k\int_0^\infty \left[1-\iota^i(\lambda)\right]\lambda f_\phi(\lambda)d\lambda$, new production of capital goods i, and capital purchased from the market $\lambda^M d^i$. Investors are not allowed to short-sell capital or bonds.

Without loss of generality, I do not allow the investors (or savers) to raise external funds to finance their purchase of capital or bonds. This assumption can be relaxed, as in Kiyotaki and Moore (2019).

A saver with existing capital k and bond b solves a similar problem, which differs from an investor's problem in that a saver cannot produce capital goods from consumption goods.

$$
\begin{aligned}
V^s(k,b;X) = &\max_{c^s,d^s,k^{s\prime},b^{s\prime},\iota^s(\lambda)} \log(c^s) + \beta\mathbb{E}\left[\pi V^i(k^{s\prime},b^{s\prime};X') + (1-\pi)V^s(k^{s\prime},b^{s\prime};X')|X\right], \\
\text{s.t.} \qquad & c^s + qd^s + q^B b^{s\prime} = b + rk + qk\int_0^\infty \iota^s(\lambda) f_\phi(\lambda)d\lambda, \\
& k^{s\prime} = k\int_0^\infty \left[1-\iota^s(\lambda)\right]\lambda f_\phi(\lambda)d\lambda + \lambda^M d^s, \\
& d^s \geq 0, \\
& b^{s\prime} \geq 0.
\end{aligned}
$$

The government. The government issues one-period real government bond B' that is sold at price q^B. The government bonds are financed by lump-sum taxes on workers T. In the crisis, the government purchases private asset D^g. The government budget constraint is

$$B + qD^g = q^B B' + T + rK^g. \tag{4}$$

Reflecting the Fed's practices in the asset purchase programs, I assume that the government lets purchased capital stay on their balance sheet and matures without selling it. In addition, I assume that the capital purchased by the government also has the average market quality λ^M, i.e., the government does not have superior information than private buyers. The government's stock of capital holding K^g follows

$$K^{g\prime} = \bar{\lambda}K^g + \lambda^M D^g. \tag{5}$$

The government sets policies on the purchase and sale of private assets D^g. It adopts a fiscal rule to ensure the inter-temporal government budget constraint:

$$T - \bar{T} = \psi \left(B - \bar{B} \right). \tag{6}$$

Market clear conditions. Denote the cumulative distribution over capital and bond holding by $\Gamma(k, b; X)$, for $j \in \{i, s\}$. By independence, the measure over capital and bond holding of investors is given by $\pi\Gamma(k, b; X)$, and that of savers is given by $(1 - \pi)\Gamma(k, b; X)$.

The aggregate demand for capital is

$$D(X) = \int d^i(k, b; X)\pi d\Gamma(k, b; X) + \int d^s(k, b; X)(1 - \pi)d\Gamma(k, b; X) + D^g.$$

The aggregate supply for capital is

$$\begin{aligned} S(X) = &\int k \left[\int_0^\infty \iota^i(k, b, \lambda; X) f_\phi(\lambda) d\lambda \right] \pi d\Gamma(k, b; X) \\ &+ \int k \left[\int_0^\infty \iota^s(k, b, \lambda; X) f_\phi(\lambda) d\lambda \right] (1 - \pi) d\Gamma(k, b; X). \end{aligned}$$

The capital market clear condition requires that

$$D(X) = S(X). \tag{7}$$

The definition of the market quality $\lambda^M(X)$ is given by

$$\begin{aligned} \lambda^M(X)S(X) = &\int k \left[\int_0^\infty \iota^i(k, b, \lambda; X)\lambda f_\phi(\lambda) d\lambda \right] \pi d\Gamma(k, b; X) \\ &+ \int k \left[\int_0^\infty \iota^s(k, b, \lambda; X)\lambda f_\phi(\lambda) d\lambda \right] (1 - \pi) d\Gamma(k, b; X). \end{aligned} \tag{8}$$

The bond market clear condition is

$$\int b^{i\prime}(k, b; X)\pi d\Gamma(k, b; X) + \int b^{s\prime}(k, b; X)(1 - \pi)d\Gamma(k, b; X) = B'. \tag{9}$$

The caiptal rental market clear condition is

$$K = \int k d\Gamma(k, b; X) + K^g.$$

Definition 1 (Recursive equilibrium) *A recursive equilibrium is (i) a set of price functions $q(X)$, $w(X)$, and $q^B(X)$ (ii) a market quality of capital $\lambda^M(X)$, (iii) a set of policy functions $\{c^j(k, b; X), d^j(k, b; X), k^{j\prime}(k, b; X), b^{j\prime}(k, b; X), \iota(k, b; X)\}_{j\in\{i,s\}}$, $H(X)$, $C^W(X)$, (iv) value functions $\{V^j(k, b; X)\}_{j\in\{i,s\}}$, (iv) a distribution $\Gamma(k, b)$ such that given government policies $B'(X)$, $D^g(X)$, $T(X)$, the following hold:*

(i) Taken the price functions as given, the policy functions solve the entrepreneurs' and workers' problem, and V^j is the value of the type-j entrepreneurs.

(ii) The market clear conditions for capital markets, bond market, and labor market hold.

(iii) The law of motion of Γ is consistent with individual decisions: $\Gamma'(\tilde{k}, \tilde{b}) = \int_{k^{i\prime}\leq\tilde{k},\, b^{i\prime}\leq\tilde{b}} \pi d\Gamma(k, b; X)$ $+ \int_{k^{s\prime}\leq\tilde{k},\, b^{s\prime}\leq\tilde{b}} (1-\pi) d\Gamma(k, b; X)$.

3 Equilibrium characterization

To describe the equilibrium, I first characterize the entrepreneurs' optimal decisions on selling and buying capital, consumption and asset holding. Then I move on to describe the aggregate economy.

Decisions to sell capital. The decisions of entrepreneurs to sell existing capital follows a simple threshold strategy:

$$\iota^i(k, b, \lambda; X) = \begin{cases} 1 & \textit{if} \quad \lambda < \max\{q(X), \lambda^M(X)\} \\ 0 & \textit{otherwise} \end{cases}$$

$$\iota^s(k, b, \lambda; X) = \begin{cases} 1 & \textit{if} \quad \lambda < \lambda^M(X) \\ 0 & \textit{otherwise} \end{cases}$$

Consider an investor. If he keeps a piece of capital indexed by λ, he gets λ units of capital goods in the subsequent period. Alternatively, he could sell the capital in the market now and gets $q(X)$ units of consumption goods, with which he could either invest and get $q(X)$ units of capital in the subsequent period or make a purchase of capital and get λ^M units of capital in the subsequent period. Consider next a saver. If he keeps a piece of capital indexed by λ, he gets λ units of capital goods in the subsequent period. Alternatively, he could sell the capital in the market now and gets $q(X)$ units of consumption goods, with which he could buy one units of capital from the market and receive $\lambda^M(X)$ efficiency units of capital in the subsequent period.

Given the threshold strategy to sell existing capital, the condition for $\lambda^M(X)$ (8) simplifies to

$$\lambda^M \left[\pi F_\phi(q) + (1-\pi)F_\phi(\lambda^M)\right] = \pi \int_0^q \lambda f_\phi(\lambda) d\lambda + (1-\pi) \int_0^{\lambda^M} \lambda f_\phi(\lambda) d\lambda. \tag{10}$$

As illustrated in Figure 2, because a saver only sells capital below market quality $\lambda^M(X)$, it follows directly that an investor must sell some capital above the market quality $\lambda^M(X)$, i.e., $q > \lambda^M$. As market price q increases, investors sell capital of higher quality, which enhances the market quality λ^M. Formally,

Proposition 1 *Given a market price of capital $q > 0$, there exists a unique value of $\lambda^M \in (0, q)$ that satisfies equation (10). Besides, the market quality λ^M increases as market price q increases.*

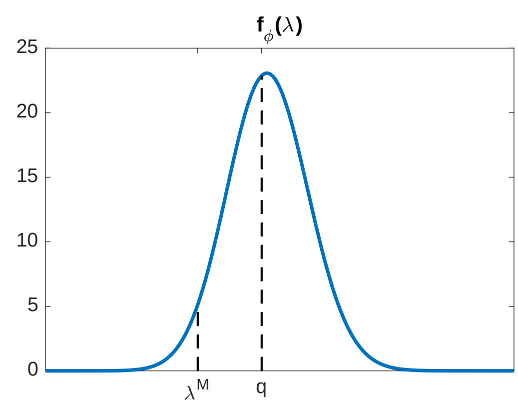

Figure 2: The thresholds for selling capital for investors and savers.

Decisions to buy capital. As $q > \lambda^M$, one can show that investors never buy capital from the market, that is, $d^i(k, b; X) = 0$. This is because with one unit of consumption goods, an investor could invest and receive 1 units of capital goods. But if he purchases capital, he could only get λ^M/q units.

It follows that an investor's problem can be reduced to the following consumption-saving and portfolio-choice problem.

$$\begin{aligned} V^i(k,b;X) = & \max_{c^i,k^{i\prime},b^{i\prime}} \log(c^i) + \beta \mathbb{E}\left[\pi V^i(k^{i\prime},b^{i\prime};X') + (1-\pi)V^s(k^{i\prime},b^{i\prime};X')|X\right], \\ \text{s.t.} \qquad & c^i + q^B b^{i\prime} + k^{i\prime} = b + \left[r + F_\phi(q)q + \int_q^\infty \lambda f_\phi(\lambda) d\lambda\right] k, \\ & b^{i\prime} \geq 0. \end{aligned} \tag{11}$$

The right-hand-side of the budget constraint is an investor's net worth, which he allocates to consumption and asset holding. Each unit of outstanding government bond is simply worth one unit of consumption good. For each unit of existing capital, an investor receives gross profit r. He then sells those pieces of quality lower than q, so those pieces are evaluated at the market price q. He keeps the remaining pieces that yield $\int_q^\infty \lambda f_\phi(\lambda)d\lambda$ efficiency units of capital and are evaluated at the replacement cost (which is one). We can define $R^i(X) = r + F_\phi(q)q + \int_q^\infty \lambda f_\phi(\lambda)d\lambda$ as the effective return on an investor's physical capital holding.

Similarly, a saver's problem is reduced to

$$\begin{aligned} V^s(k,b;X) = & \max_{c^s,k^{s\prime},b^{s\prime}} \log(c^s) + \beta \mathbb{E}\left[\pi V^i(k^{s\prime},b^{s\prime};X') + (1-\pi)V^s(k^{s\prime},b^{s\prime};X')|X\right], \\ \text{s.t.} \qquad & c^s + q^B b^{s\prime} + \frac{q}{\lambda^M} k^{s\prime} = b + \left[r + F_\phi(\lambda^M)q + \frac{q}{\lambda^M}\int_{\lambda^M}^\infty \lambda f_\phi(\lambda) d\lambda\right] k, \\ & b^{s\prime} \geq 0. \end{aligned} \tag{12}$$

We can define $R^s(X) = r + F_\phi(\lambda^M)q + \frac{q\int_{\lambda^M}^\infty \lambda f_\phi(\lambda)d\lambda}{\lambda^M}$ as the return on a saver's physical capital holding. Adverse selection causes three differences between an investor's problem and a saver's problem. First, a saver's replacement cost of capital $\frac{q}{\lambda^M}$ is larger than that of an investor. Second, one can easily show that a saver's return on capital $R^s(X)$ is greater than that of an investor $R^i(X)$. Third, the saver sells capital of quality up to λ^M instead of q.

The following two propositions characterize the solutions to the investor's and saver's problems.

Proposition 2 (Policy functions of investors) *The policy functions for investors satisfy the following.*

(i) $c^i(k,b;X) = (1-\beta)n^i$*, where* $n^i = b + R^i k$ *is the net worth of an investor.*

(ii) Define $\phi^i(k,b;X) = \frac{k^{i\prime}}{q^B b^{i\prime} + k^{i\prime}}$ *the portfolio weight of an investor on capital. The Euler equation of an investor satisfies*

$$q^B \geq \mathbb{E}\left[\pi \frac{1}{\phi^i R^i(X') + (1-\phi^i)\frac{1}{q^B}} + (1-\pi)\frac{1}{\phi^i R^s(X') + (1-\phi^i)\frac{1}{q^B}} | X\right], \tag{13}$$

where the inequality holds iff $\phi^i = 1$*.*

Proposition 2 shows that all investors, independent of their outstanding asset holding (k,b), all choose the same portfolio weight ϕ^i. This property is due to the assumption of CRRA utility function. It greatly simplifies the analysis, because now I only need to keep track of a single Euler equation for all investors, instead of one for each individual investor. Besides, aggregation across investors also becomes straight forward. Similarly, the following result holds for savers.

Proposition 3 (Policy functions of savers) *The policy functions for savers satisfy the following.*

(i) $c^s(k,b;X) = (1-\beta)n^s$*, where* $n^s = b + R^s k$ *is the net worth of a saver.*

(ii) Define $\phi^s(k,b;X) = \frac{qk^{s\prime}/\lambda^M}{q^B b^{s\prime} + qk^{s\prime}/\lambda^M}$ *the portfolio weight of a saver on capital. The Euler equation of a saver satisfies*

$$q^B \geq \mathbb{E}\left[\pi \frac{1}{\phi^s \frac{\lambda^M}{q} R^i(X') + (1-\phi^s)\frac{1}{q^B}} + (1-\pi)\frac{1}{\phi^s \frac{\lambda^M}{q} R^s(X') + (1-\phi^s)\frac{1}{q^B}} | X\right], \tag{14}$$

where the inequality holds iff $\phi^s = 1$*.*

In this model, entrepreneurs face idiosyncratic risks. Since $R^s(X) > R^i(X)$, the return on physical is lower exactly when an entrepreneur has the opportunity to invest and needs the liquidity the most. This is a source of inefficiency in this model. Besides, $R^s(X) > R^i(X)$ implies that $c^s(k,b;X) > c^i(k,b;X)$, i.e., a saver with the same (k,b) as an investor will consume more and hold fewer assets. In other words, entrepreneurs also face idiosyncratic consumption risks.

In this economy, investors are less willing to hold government bonds than savers. They take advantage of their investment opportunities to produce new capital goods. In particular, I can show that in the steady state of the aggregate economy, investors do not hold any government bonds.

Proposition 4 *In the steady state where the capital market is active ($q > 0$), investors do not hold government bonds, i.e., $\bar{\phi}^i = 1$.*

In the economy out of the steady state, we assume that $\phi^i = 1$ always holds, solve the model and verify that it is true.[3]

The aggregate economy. Given the linearity in the policy functions of entrepreneurs, I do not need to keep track of the distribution of asset holdings as aggregate state variables. The endogenous state variables in this model are K, K^g, and B. Aggregate investment is

$$I = \pi \left[\beta \left(B + R^i K^p \right) - K^p \int_q^{\infty} \lambda f_{\phi}(\lambda) d\lambda \right], \tag{15}$$

where $K^p = K - K^g$ is the private holding of capital. The evolution of aggregate capital K is given by

$$K' = \bar{\lambda} K + I. \tag{16}$$

B' is proportional to the aggregate net worth of savers:

$$B' = \frac{\beta (1-\pi)(1-\phi^s)(B + R^s K^p)}{q^B}. \tag{17}$$

The evolution of K^g is characterized by (5). The capital market clear condition is given by

$$\frac{1}{q} \beta (1-\pi) \phi^s (B + R^s K^p) - \frac{\int_{\lambda^M}^{\infty} \lambda f(\lambda) d\lambda}{\lambda^M} K^p + D^g = \left[\pi F(q) + (1-\pi) F(\lambda^M) \right] K^p. \tag{18}$$

The social resource constraint is

$$\underbrace{(1-\beta)\pi \left(B + R^i K^p \right)}_{\text{consumption of investors}} + \underbrace{(1-\beta)(1-\pi)(B + R^s K^p)}_{\text{consumption of savers}} + \underbrace{(1-\alpha) Y - T}_{\text{consumption of workers}} + I = Y. \tag{19}$$

[3] In the steady state, if the government provides sufficient liquidity by issuing sufficiently large amount of debt B, then adverse selection can be eliminated. No investor needs to sell capital, because their holding of bonds provides enough liquidity for making investment. Consequently, the capital market shuts down. There is no liquidity premium for government bond and $q^B = \beta$. Investors therefore are indifferent between buying government bonds and producing capital and it can be that $\bar{\phi}^i < 1$. Besides, all idiosyncratic risks are eliminated. Whether this is the first best depends on the social welfare function and the welfare weights a planner puts on workers and entrepreneurs, as a larger stock of bond B is backed by lump-sum taxes levied on workers.

The gross profit of capital is

$$r = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha+\epsilon}} A^{\frac{1+\epsilon}{\alpha+\epsilon}} K^{\frac{\epsilon(\alpha-1)}{\alpha+\epsilon}}. \tag{20}$$

The aggregate economy can be characterized by quantities K', $K^{g\prime}$, B', ϕ^i, I, ϕ^s and prices q, q^B, average quality of capital λ^M, such that given bond supply policy T, D^g, equations (5), (10), (13), (14), (15), (16), (17), (18), (19) hold.

3.1 Equivalence with an economy with symmetric information and taxes and subsidies

As discussed before, asymmetric information has two effects on the model economy. First, it induces a wedge between the replacement costs of capital of investors (1) and savers (q/λ^M), because buyers cannot avoid buy pieces of capital of quality below the price of capital. It is as if buyers are paying a tax on their purchases. Second, information asymmetry allows sellers to sell pieces of capital of quality below the price of capital, which is like a subsidy.

To better illustrate the role played by asymmetric information in the model, in the spirit of Kurlat (2013) I show the equivalence of our model economy with an economy with symmetric information and taxes. Consider an economy where all information is public. In this economy, entrepreneurs can trade efficiency units of capital in the capital market at price p. At the same time, the government imposes three types of taxes or subsidies: (i) a tax τ on the purchases of capital; (ii) a subsidy η^i on each efficiency unit of capital held by investors; (iii) a subsidy η^s on each efficiency unit of capital held by savers. An investor in this economy solves the following problem

$$\begin{aligned}
V^i(k,b;X) = &\max_{c^i,d^i,s^i,k^{i\prime},b^{i\prime},i} \log(c^i) + \beta\mathbb{E}\left[\pi V^i(k^{i\prime},b^{i\prime};X') + (1-\pi)V^s(k^{i\prime},b^{i\prime};X')|X\right], \\
\text{s.t.} \quad & c^i + (p+\tau)d^i + q^B b^{i\prime} + i = b + rk + ps^i + \eta^i k, \\
& k^{i\prime} = \bar{\lambda}k - s^i + i + d^i, \\
& d^i \geq 0, \\
& b^{i\prime} \geq 0, \\
& 0 \leq s^i \leq \bar{\lambda}k,
\end{aligned}$$

where d^i is the purchase of capital from the capital market, and s^i is the sales of existing capital.

An saver with existing capital k and bond b solves the problem

$$
\begin{aligned}
V^s(k,b;X) = &\max_{c^s,d^s,s^s,k^{s\prime},b^{s\prime}} \log(c^s) + \beta\mathbb{E}\left[\pi V^i(k^{s\prime},b^{s\prime};X') + (1-\pi)V^s(k^{s\prime},b^{s\prime};X')|X\right], \\
\text{s.t.} \qquad & c^s + (p+\tau)d^s + q^B b^{s\prime} = b + rk + ps^s + \eta^s k, \\
& k^{s\prime} = \bar{\lambda}k - s^s + d^s, \\
& d^s \geq 0, \\
& b^{s\prime} \geq 0, \\
& 0 \leq s^s \leq \bar{\lambda}k.
\end{aligned}
$$

The government also purchases capital. Denote the net purchase by D^g_{SI}, where subscript SI represents symmetric information. The government budget constraint is

$$
B + (p+\tau)D^g_{SI} + \left[\pi\eta^i + (1-\pi)\eta^s\right]K^p = q^B B' + T_{SI} + rK^g + \tau S^i.
$$

Where S^i is the sales of capital by the investors.[4] The government capital holding evolves as

$$
K^{g\prime} = \bar{\lambda}K^g + D^g_{SI}.
$$

The following proposition shows that if the tax τ and subsidies η^i and η^s are chosen in a specific way, real allocations and the government bond price are the same in this symmetric information economy and the asymmetric information economy in the previous section.

Proposition 5 *Suppose $\tau(X) = \frac{q}{\lambda^M} - 1$, $\eta^i(X) = F_\phi(q)q - \int_0^q \lambda f_\phi(\lambda)d\lambda$, $\eta^s(X) = F_\phi(\lambda^M)q - \frac{q}{\lambda^M}\int_0^{\lambda^M} \lambda f_\phi(\lambda)d\lambda$, $D^g_{SI}(X) = \lambda^M D^g$, $T_{SI} = T$, where q and λ^M are the equilibrium values in the asymmetric information economy. Then (i) the real allocations and bond price of the symmetric-information economy and the asymmetric information economy are identical; (ii) in the symmetric-information economy, $p = 1$; (iii) the tax τ and subsidies η^i, η^s are revenue neutral, i.e., $\tau S^i = \left[\pi\eta^i + (1-\pi)\eta^s\right]K^p$.*

[4]Savers buy capital without selling capital it in this symmetric-information economy, since when $\tau > 0$ it is not optimal to buy and sell at the same time.

In the absence of the tax τ and subsidies η^i, η^s, the symmetric-information economy would feature first-best investment. The market price of capital $p = 1$, which equals the cost of investment. The investors are able to sell enough capital to finance first-best investment. Asymmetric information induces tax τ and subsidies η^i, η^s in a specific manner. τ is equal to the difference between the replacement costs of savers and investors in the asymmetric-information economy. η^i and η^s equal the difference sales revenues and the efficiency units of capital. In response to aggregate shocks, the severity of adverse selection in the symmetric information economy will endogenously fluctuate, which can be completely captured by endogenous fluctuations in τ, η^i and η^s as defined in Proposition 5.

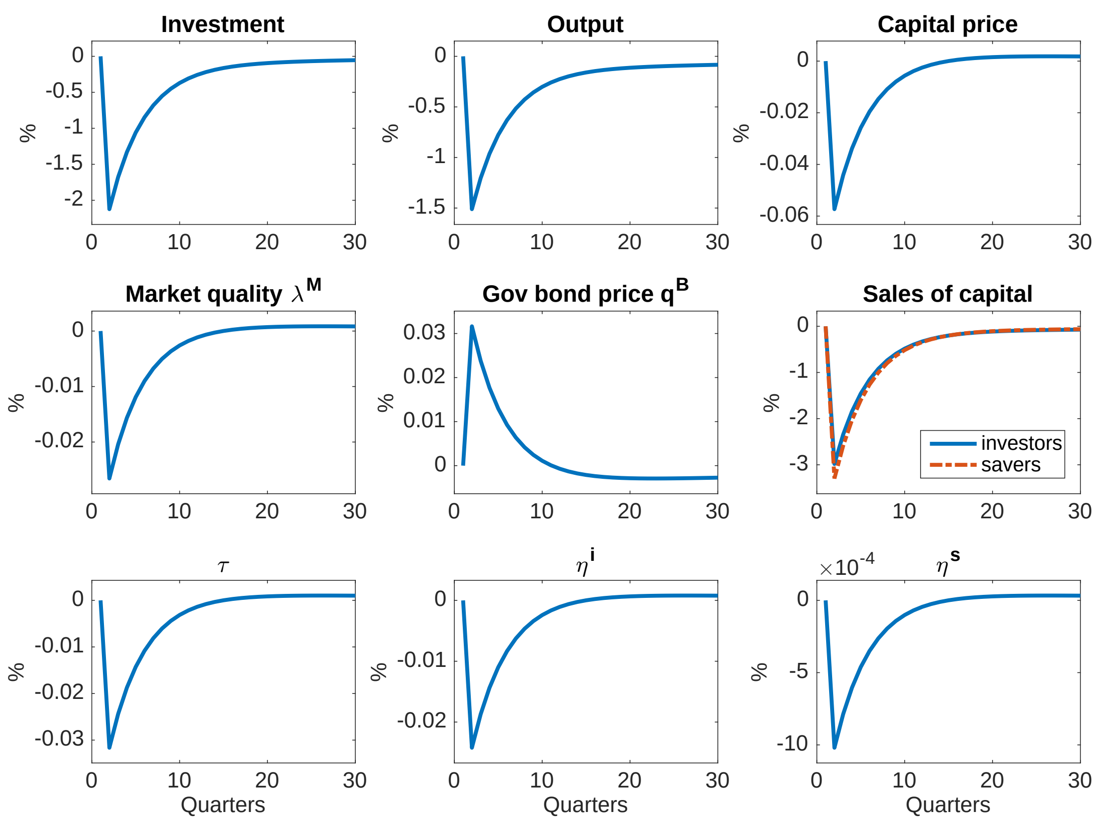

Figure 3: Responses to a negative 1% shock to A. I assume that A follows an AR(1) process with autoregressive coefficient of 0.78. The figure shows percentage deviations from steady-state value.

To illustrate the model behavior, I simulate the model and present the impulse responses to shocks. Figure 3 shows the responses to negative one percent shock to the TFP. As the shock dampens investment, the willingness to sell existing capital declines, which results in falls in the capital price and the market quality λ^M and deteriorates the adverse selection. Greater illiquidity of physical capital causes the sales of by both investors and savers decrease. As investors lack the liquidity needed to make investment, contraction in aggregate investment is amplified. At the same time, savers hoard liquid government bonds, which pushes down the government bond yield.

The bottom three panels show the dynamics of implied taxes and subsidies in response to a negative TFP shock. τ declines because as capital price q and market quality λ^M both decline, the ratio $\frac{q}{\lambda^M}$ gets smaller and closer to one.[5] It implies that the replacement cost of capital between investors and savers become closer. However, it does not mean that adverse selection is alleviated by a negative TFP shock. Instead, the subsidies received by investors and savers both fall, which means scarcer liquidity for both investors and savers.

Figure 4 shows the responses to a one-time shock to the dispersion of capital quality ϕ. In the initial period, ϕ doubles from 1.73% to 3.46% and then declines at the rate $\rho_\phi = 0.5$. The shock has recessionary effects on aggregate investment and production. After the increase in the dispersion of capital quality, adverse selection becomes more severe. As the capital price falls, the market quality λ^M also declines. Consequently, the fraction of capital sold by investors and savers both take a hit, and the liquidity of investors becomes scarcer. At this point, savers hoard liquid government bonds, which pushes down the government bond yield.

The bottom three panels show the dynamics of implied taxes and subsidies in response to a shock to ϕ. The increase in ϕ induces a larger wedge between the cost of capital of savers and investors $\tau = \frac{q}{\lambda^M} - 1$. At the same time, a more severe adverse selection increases the subsidies η^i and η^s on the capital sold in the market.

3.2 Liquidity facilities

I analyze the effect of government's purchase of private assets financed by issuing liquid government bonds. The blue line in Figure 5 shows the economy's responses to the government's purchase. In period 0, the economy is in a steady state with no government purchase $D^g = 0$. In period 1, the government purchase assets amounts to 5% of GDP, and then the purchase declines at rate 0.8

[5] One can prove that holding ϕ constant, a lower capital price q causes $\frac{q}{\lambda^M}$ to decline.

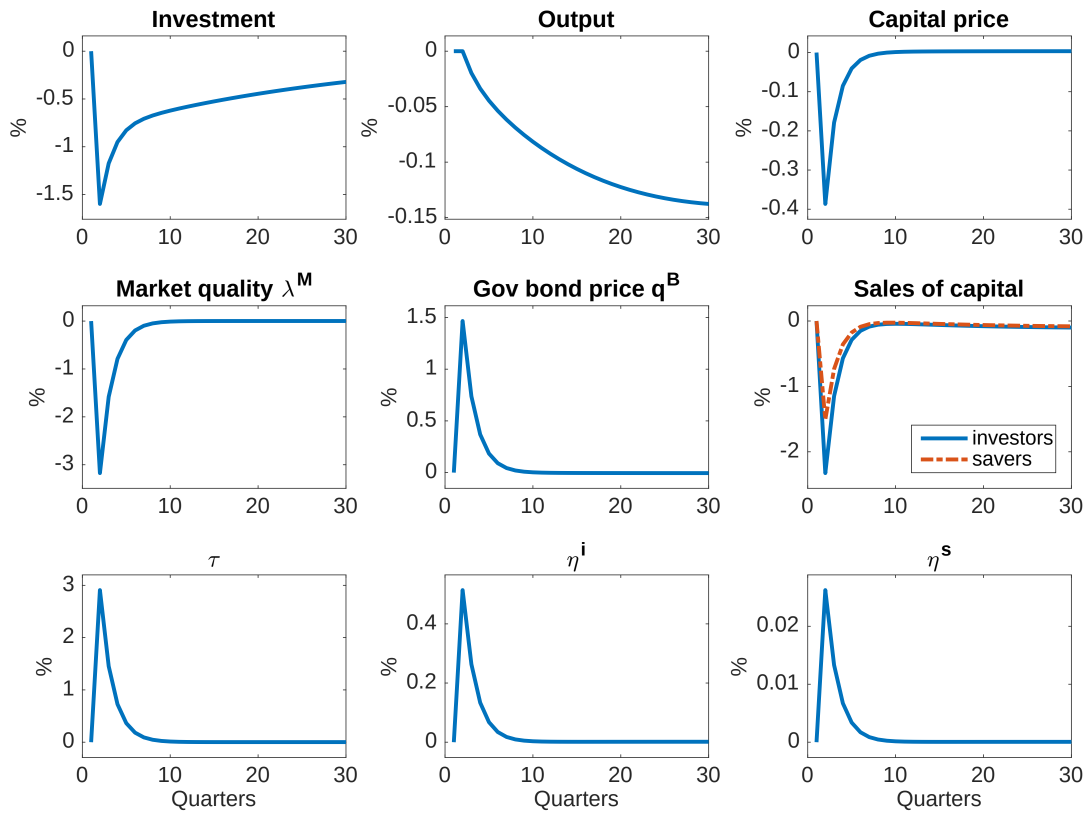

Figure 4: Responses to a one-time shock to ϕ. The shock in the initial period doubles the standard deviation of capital quality. In the initial period, ϕ doubles from 1.73% to 3.46% and then declines at the rate $\rho_\phi = 0.5$. The figure shows percentage deviations from steady-state value.

in subsequent periods: [6] The government finances the purchase through debt and taxes as in the fiscal rule (6).

Due to the government's purchase, capital price rises, and investors and savers sell a larger share of their capital to fund investment. The government bond price initially declines because of higher issuance of debt. After a few periods, as savers' net worth increases, they have a higher demand for government bonds (as well as capital), and therefore the bond price rises consequently.

The bottom three panels show the dynamics of implied taxes and subsidies. A higher capital price increases the wedge between the cost of capital of savers and investors $\tau = \frac{q}{\lambda^M} - 1$. At the same time, subsidies η^i and η^s increase as a result of higher capital price and market quality.

[6] I assume that government does not sell the assets. Rather, the government lets the assets stay on their balance sheet and gradually depreciate (mature).

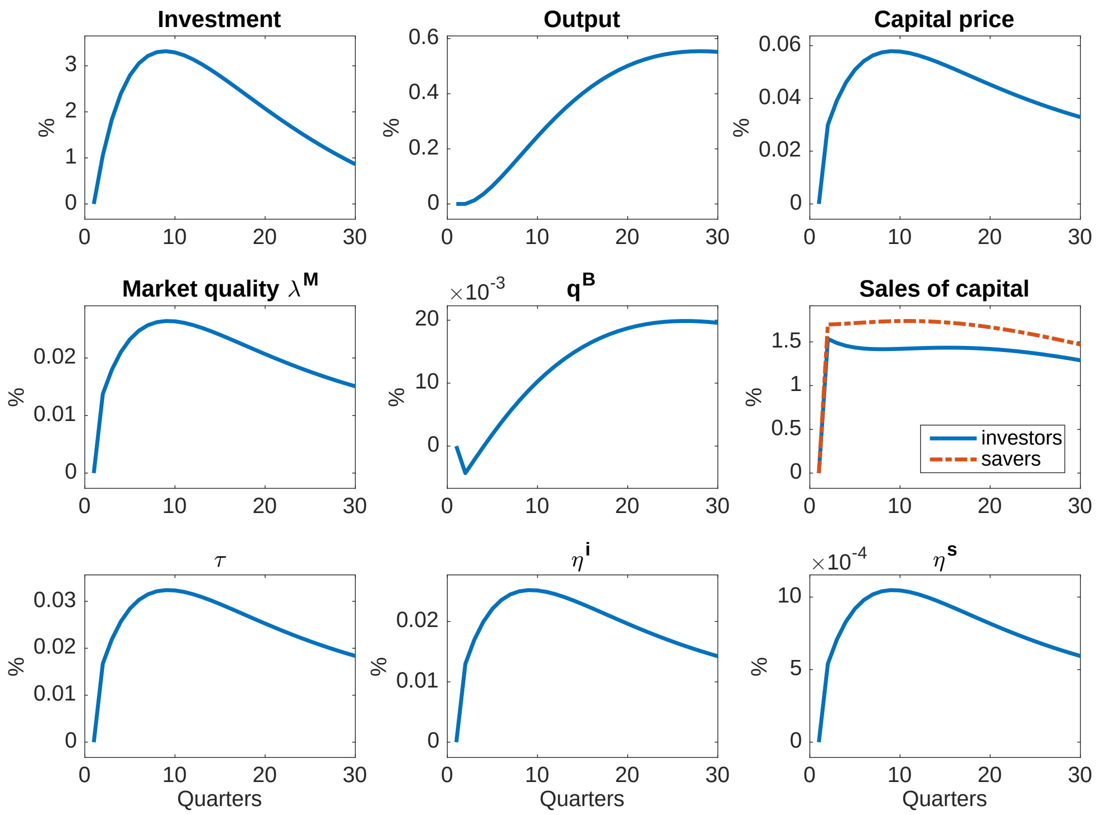

Figure 5: Responses to government's purchase of assets (D^g). In period 1, the government purchase assets amounts to 5% of GDP, and then the purchase declines at rate 0.8 in subsequent periods.

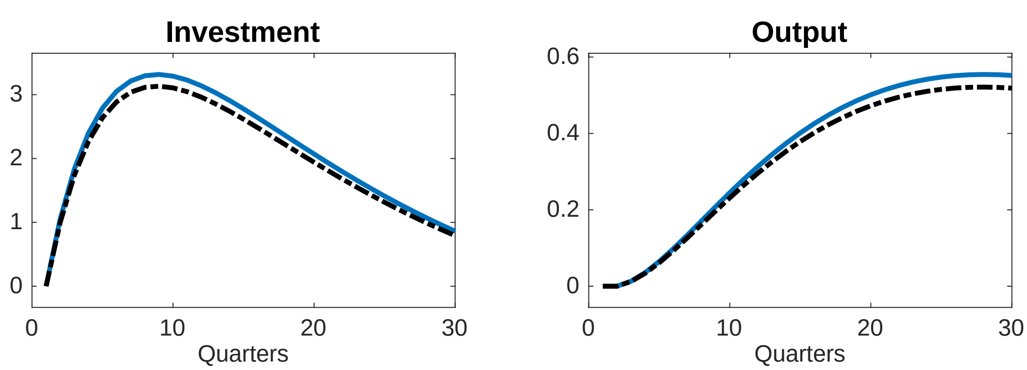

Figure 6: Responses to government's purchase of assets (D^g). In period 1, the government purchase assets amounts to 5% of GDP, and then the purchase declines at rate 0.8 in subsequent periods.

Due to the no-short-selling constraint, government purchases have real effects even in the absence of asymmetric information. In order to single out the role played by asymmetric information, I study the government's purchase in the equivalence economy with symmetric information when τ, η^i, and η^s are stay at their steady-state values. That is, government purchases have no effect on the severity of adverse selection. As shown in the black-dotted line in Figure 5, the government's purchase becomes less effective, although quantitatively the difference is not very large.

4 Crisis experiment

4.1 Calibration

I consider two one-time aggregate shocks. A shock to the TFP (A) and a shock to the dispersion of capital quality (ϕ). I assume $AR(1)$ processes for A and ϕ. I estimate $\rho_A = 0.78$ from the non-utilization-adjusted TFP series constructed by Fernald (2014). I set $\rho_\phi = 0.5$, so the shock diminishes after a year. In the crisis experiment, I feed the model with a one-time -4% TFP shock, and a one-time shock to ϕ equal to 15% (see Figure 7). The choice is ϕ is such that the maximum decline in aggregate investment matches that in the NIPA data.

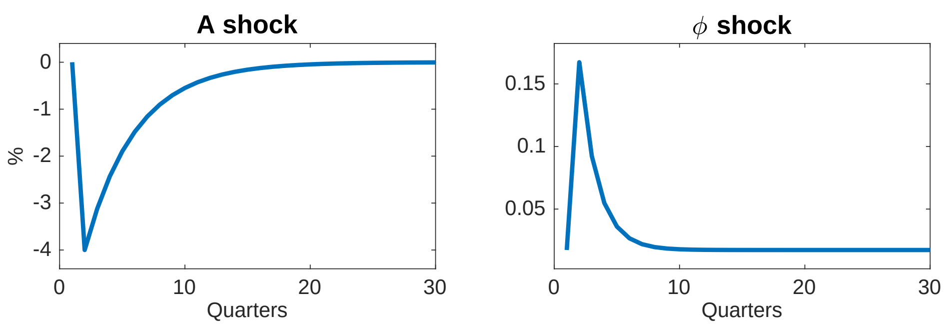

Figure 7: The TFP shock and ϕ shock in the crisis experiment. $\rho^A = 0.78$, $\rho^\phi = 0.5$.

The parameters in the numerical exercise is shown in Table 1. The model is calibrated at the quarterly frequency. I set the average quality of capital $\bar{\lambda}$ such that the annual depreciation rate equals 10%. The steady-state debt-to-GDP is set to 40%, consistent with Federal debt held by the public before the crisis. I use a small coefficient $\psi = 0.05$ for the Taylor rule. That is, taxes slowly adjust to the government purchase of private assets after intervention, so that government debt finances most of the intervention in the short run.

Two parameters central to the severities of financial frictions are the share of investors and the baseline dispersion of capital quality $\bar{\phi}$. I calibrate them to meet two targets. First, the steady-state liquidity premium of liquid government bonds is 0.46%,[7] consistent with the estimates by Krishnamurthy and Vissing-Jorgensen (2012) of the average liquidity premium from 1926–2008. Second, the steady-state share of liquid assets on entrepreneurs' balance sheet $\frac{\bar{q}^B \bar{B}}{\bar{q}^B \bar{B} + \bar{q}\bar{K}^p}$ is 15%, consistent with that of the U.S. financial institutions from the US Flow of Funds (Radde, 2015).

The government's asset purchase follows:

$$D^{g\prime} = \rho^D D^g + e^D.$$

I set ρ_D to 0.8 and the size of the initial shock to 3% of GDP. It generates the result that the government holding of capital reaches 10% of GDP after one year, consistent with the increase on the asset side of the Fed's balance sheet after the collapse of Lehman Brothers. I compute the non-linear perfect foresight path after the arrival of shocks.

Table 1: Parameters

		Value	Target/Source
Preference and technology			
Household discount factor	β	0.990	
Inverse Frisch elasticity	ϵ	1.000	in line with Chetty et al. (2011)
Capital share of output	α	0.330	
Average quality of capital	$\bar{\lambda}$	0.975	Average annual depreciation rate 10%
Financial friction			
Probability of investing	π	0.040	
Baseline std of capital quality	$\bar{\phi}$	0.0173	calibrated
Policy			
SS debt-to-GDP ratio	$\frac{\bar{B}}{\bar{Y}}$	0.400	Federal debt held by the public before crisis
Fiscal rule coefficient	ψ	0.050	
Shocks			
Serial correlation of A	ρ_A	0.780	Estimated using Fernald (2014)
Serial correlation of ϕ	ρ_ϕ	0.500	

[7] The steady-state annualized liquidity premium is defined as $4(1/\beta - 1/\bar{q}^B)$. It is defined as the difference between the interest rates paid on government bond and an asset without liquidity value (and therefore the interest rate is $1/beta$.)

Figure 8 compares the responses of investment, output, and consumption to the data.[8] The model matches the size of contraction in investment by design, as the size of ϕ shock is chosen to generate the contraction in investment in data. The model also generates a decline in aggregate output comparable to that in the data. However, it understates the fall in aggregate consumption.

The solid line in Figure 9 shows that the contraction in real activity is accompanied by deteriorating liquidity. The model generates a 7.9% decline in capital prices, and the sales of capital by investors and savers crash by 32% and 26%, respectively. At the same time, government bond prices increases sharply, inconsistent with data. The reason is that we need a large ϕ shock to generate the decline in investment, and q^B is very responsive to ϕ shock.

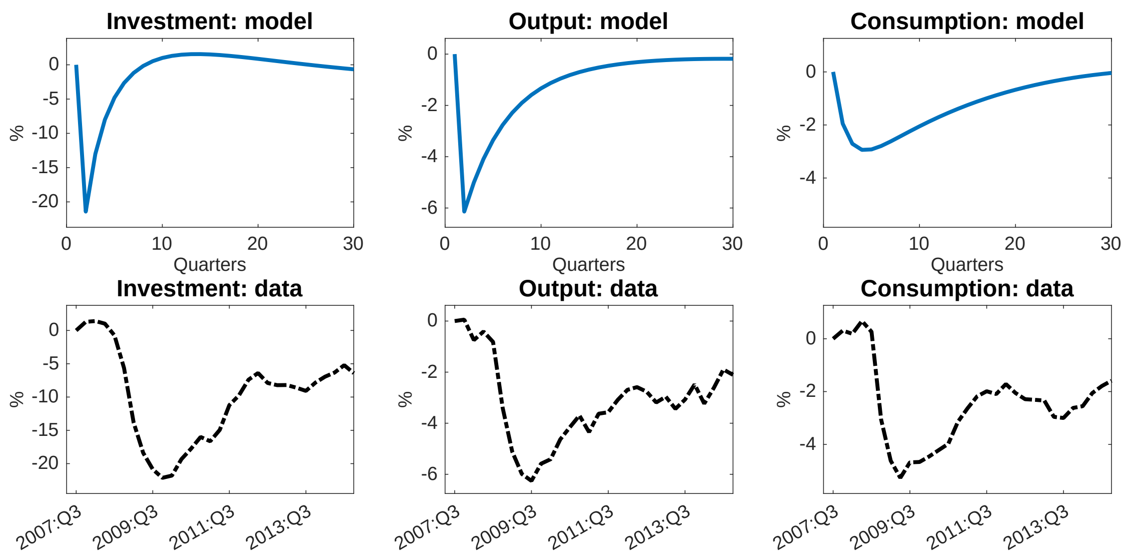

Figure 8: Responses of investment, output, and consumption in the crises experiment.

What would happen without the liquidity facilities? The black dashed line in Figure 9 shows the result for the same A and ϕ shocks. Without liquidity facilities, the initial loss in investment would be 2.3% larger. The initial loss in output would be the same because it is only affected by the TFP shock. But its recovery, as well as the recovery of investment becomes more sluggish without the liquidity facilities.

[8]The data series are from the NIPA. They correspond to the Gross Private Non-Residential Fixed Investment, Gross Domestic Product, Personal Consumption Expenditure.

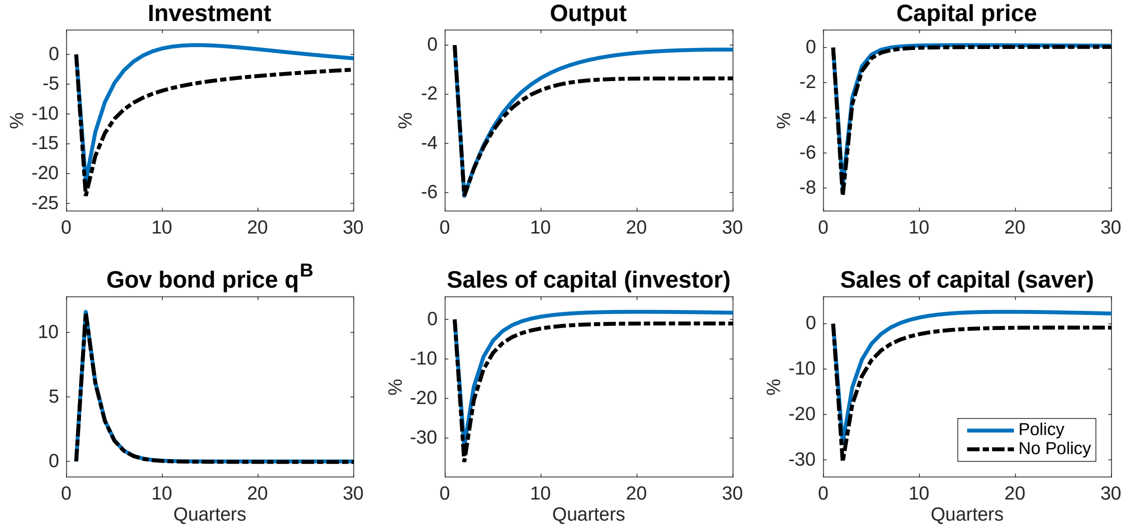

Figure 9: Responses to government's purchase of assets (D^g).

Table 2 shows that the 10-year average losses in investment would be 5 times as large, and the average losses in output would almost double. Capital price would tank by 0.5% more in the initial period, and sales of capital by investors and savers would have declind by 6% and 4.5% more, respectively. These results suggest that the quantitative effects of liquidity facilities are sizable.

Table 2: Losses in output and investment

	Policy		No policy	
	Initial-period	10-yr average	Initial-period	10-yr average
Investment	-21.4%	-4.9%	-23.7%	-19.8%
Output	-6.1%	-3.9%	-6.1%	-7.2%

5 Concluding remarks

In this paper, I propose a general equilibrium model with asymmetric information in asset quality. The equilibrium features shortage in liquidity and suboptimal investment, the severity of which endogenously responds to aggregate shocks. Government liquidity facilities that issue liquid government bonds to purchase illiquid private assets can alleviate the adverse selection and relax financing constraints. I find large quantitative effects of liquidity facilities on credit market conditions, aggregate investment and output in the Great Recession. An interesting next step is to investigate whether asset purchase programs creates a moral hazard problem by incentivizing entrepreneurs to over-invest in illiquid private assets ex ante without holding enough liquid assets.

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Appendix

A Proof of Proposition 1

Define the difference between the RHS and LHS of equation (10) by

$$G(\lambda^M, q) = \left[\pi \int_0^q \lambda f_\phi(\lambda) d\lambda + (1-\pi) \int_0^{\lambda^M} \lambda f_\phi(\lambda) d\lambda\right] - \lambda^M \left[\pi F_\phi(q) + (1-\pi) F_\phi(\lambda^M)\right].$$

If $q > 0$, then

$$G(q, q) = \int_0^q \lambda f(\lambda) d\lambda - qF(q) < 0,$$

$$G(0, q) = \pi \int_0^q \lambda f(\lambda) d\lambda > 0.$$

$$\frac{\partial G}{\partial \lambda^M} = -\left[\pi F(q) + (1-\pi) F(\lambda^M)\right] < 0.$$

Therefore, there exists a unique $\lambda^M(q) \in (0, q)$ such that equation (10) holds. One can also show that

$$\frac{\partial G}{\partial q} = \pi f(q)(q - \lambda^M) > 0.$$

Therefore,

$$\lambda^{M\prime}(q) = \frac{-\frac{\partial G}{\partial q}}{\frac{\partial G}{\partial \lambda^M}} > 0.$$

B Proof of Proposition 2

The solution to the investor's problem satisfies the following conditions

$$\text{Euler for } k^{i\prime}: \quad \frac{1}{c^i} = \beta \mathbb{E}\left(\pi \frac{R^{i\prime}}{c^{i\prime}} + (1-\pi) \frac{R^{s\prime}}{c^{s\prime}} \middle| X\right),$$

$$\text{Euler for } b^{i\prime}: \quad \frac{q^B}{c^i} = \beta \mathbb{E}\left(\pi \frac{1}{c^{i\prime}} + (1-\pi) \frac{1}{c^{s\prime}} \middle| X\right) + \gamma_b^i.$$

γ_b^i is the multiplier on $b^{i\prime} \geq 0$. With a slight abuse of notation, I use $c^{i\prime}$ (or $c^{s\prime}$) to denote the next-period consumption of the investor if she becomes an investor (or a saver) next period.

To prove proposition 2, it is equivalent to prove that $c^i(k, b; X) = (1-\beta)n^i$, $k^{i\prime}$, and $b^{i\prime}$ that satisfy the two Euler equations above also satisfy equation (13), and vice versa.

One can show that

$$
\begin{aligned}
c^{i\prime} &= (1-\beta)n' = (1-\beta)\left(R^{i\prime}k^{i\prime} + b^{i\prime}\right) = (1-\beta)\left[R^{i\prime}\phi^i\beta n^i + (1-\phi^i)\beta n^i \frac{1}{q^B}\right] \\
&= (1-\beta)\beta\left[\phi^i R^{i\prime} + (1-\phi^i)\frac{1}{q^B}\right] n^i.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
c^{s\prime} &= (1-\beta)n' = (1-\beta)\left(R^{s\prime}k^{i\prime} + b^{i\prime}\right) = (1-\beta)\left[R^{s\prime}\phi^i\beta n^i + (1-\phi^i)\beta n^i \frac{1}{q^B}\right] \\
&= (1-\beta)\beta\left[\phi^i R^{s\prime} + (1-\phi^i)\frac{1}{q^B}\right] n^i.
\end{aligned}
$$

Substituting for c^i, $c^{i\prime}$ and $c^{s\prime}$, the above two Euler equations are equivalent to

$$
q^B = \mathbb{E}\left[\pi \frac{1}{\phi^i R^{i\prime} + (1-\phi^i)\frac{1}{q^B}} + (1-\pi)\frac{1}{\phi^i R^{s\prime} + (1-\phi^i)\frac{1}{q^B}} | X\right] - \frac{\gamma_b^i}{n^i}, \tag{21}
$$

$$
1 = \mathbb{E}\left[\pi \frac{R^{i\prime}}{\phi^i R^{i\prime} + (1-\phi^i)\frac{1}{q^B}} + (1-\pi)\frac{R^{s\prime}}{\phi^i R^{s\prime} + (1-\phi^i)\frac{1}{q^B}} | X\right]. \tag{22}
$$

So I have shown that $c^i(k, b; X) = (1-\beta)n^i$, $k^{i\prime}$, and $b^{i\prime}$ that satisfy the two Euler equations above also satisfy equation (13) (which is equivalent to equation (21)). To show that allocations that satisfy (13) also satisfy (21) and (22), I discuss the following two cases.

Case 1: $\phi^i = 1$ and $\gamma_b^i > 0$. In this case, equation (22) trivially holds:

$$
1 = \mathbb{E}\left[\pi \frac{R^{i\prime}}{R^{i\prime}} + (1-\pi)\frac{R^{s\prime}}{R^{s\prime}} | X\right] = 1.
$$

Case 2: $\phi^i < 1$ and $\gamma_b^i = 0$. In this case, I will compute (21)$\times(1-\phi^i)/q^B$+(22)$\times\phi^i$ and show that it trivially holds. The LHS is

$$
(1-\phi^i)/q^B \times q^B + \phi^i = 1.
$$

The RHS is

$$\mathbb{E}\left[\pi\frac{\frac{1-\phi^i}{q^B}}{\phi^i R^{i\prime}+(1-\phi^i)\frac{1}{q^B}}+(1-\pi)\frac{\frac{1-\phi^i}{q^B}}{\phi^i R^{s\prime}+(1-\phi^i)\frac{1}{q^B}}|X\right]$$
$$+\mathbb{E}\left[\pi\frac{\phi^i R^{i\prime}}{\phi^i R^{i\prime}+(1-\phi^i)\frac{1}{q^B}}+(1-\pi)\frac{\phi^i R^{s\prime}}{\phi^i R^{s\prime}+(1-\phi^i)\frac{1}{q^B}}|X\right]=1.$$

Therefore, as long as (21) holds, equation (22) also holds. So I have shown that allocations that satisfy (13) also satisfy (21) and (22) .

C Proof of Proposition 3

The proof is very similar to that of Proposition 2. The solution to the saver's problem satisfies the following conditions

$$\text{Euler for } k^{s\prime}: \quad \frac{q}{\lambda^M c^s}=\beta\mathbb{E}\left(\pi\frac{R^{s\prime}}{c^{i\prime}}+(1-\pi)\frac{R^{s\prime}}{c^{s\prime}}\middle| X\right),$$
$$\text{Euler for } b^{s\prime}: \quad \frac{q^B}{c^s}=\beta\mathbb{E}\left(\pi\frac{1}{c^{i\prime}}+(1-\pi)\frac{1}{c^{s\prime}}\middle| X\right)+\gamma_b^s.$$

γ_b^s is the multiplier on $b^{s\prime}\geq 0$. With a slight abuse of notation, I use $c^{i\prime}$ (or $c^{s\prime}$) to denote the next-period consumption of the investor if she becomes an investor (or a saver) next period.

To prove proposition 3, it is equivalent to prove that $c^s(k,b;X)=(1-\beta)n^s$, $k^{s\prime}$, and $b^{s\prime}$ that satisfy the two Euler equations above also satisfy equation (13), and vice versa.

One can show that

$$c^{i\prime}=(1-\beta)n'=(1-\beta)\left(R^{i\prime}k^{s\prime}+b^{s\prime}\right)=(1-\beta)\left[R^{i\prime}\frac{\lambda^M}{q}\phi^s\beta n^s+(1-\phi^s)\beta n^s\frac{1}{q^B}\right]$$
$$=(1-\beta)\beta\left[\phi^s\frac{\lambda^M}{q}R^{i\prime}+(1-\phi^s)\frac{1}{q^B}\right]n^s.$$

Similarly,

$$c^{s\prime}=(1-\beta)n'=(1-\beta)\left(R^{s\prime}k^{s\prime}+b^{s\prime}\right)=(1-\beta)\left[R^{s\prime}\frac{\lambda^M}{q}\phi^s\beta n^s+(1-\phi^s)\beta n^s\frac{1}{q^B}\right]$$
$$=(1-\beta)\beta\left[\phi^s\frac{\lambda^M}{q}R^{s\prime}+(1-\phi^s)\frac{1}{q^B}\right]n^s.$$

Substituting for c^s, $c^{i\prime}$ and $c^{s\prime}$, the above two Euler equations are equivalent to

$$q^B = \mathbb{E}\left[\pi\frac{1}{\phi^s\frac{\lambda^M}{q}R^{i\prime}+(1-\phi^s)\frac{1}{q^B}}+(1-\pi)\frac{1}{\phi^s\frac{\lambda^M}{q}R^{s\prime}+(1-\phi^s)\frac{1}{q^B}}|X\right]-\frac{\gamma_b^s}{n^s}, \tag{23}$$

$$\frac{q}{\lambda^M} = \mathbb{E}\left[\pi\frac{R^{i\prime}}{\phi^s\frac{\lambda^M}{q}R^{i\prime}+(1-\phi^s)\frac{1}{q^B}}+(1-\pi)\frac{R^{s\prime}}{\phi^s\frac{\lambda^M}{q}R^{s\prime}+(1-\phi^s)\frac{1}{q^B}}|X\right]. \tag{24}$$

So I have shown that $c^s(k,b;X)=(1-\beta)n^s$, $k^{s\prime}$, and $b^{s\prime}$ that satisfy the two Euler equations above also satisfy equation (14) (which is equivalent to equation (23)). To show that allocations that satisfy (14) also satisfy (23) and (24), I discuss the following two cases.

Case 1: $\phi^s = 1$ and $\gamma_b^s > 0$. In this case, equation (24) trivially holds:

$$\frac{q}{\lambda^M} = \mathbb{E}\left[\pi\frac{R^{i\prime}}{\frac{\lambda^M}{q}R^{i\prime}}+(1-\pi)\frac{R^{s\prime}}{\frac{\lambda^M}{q}R^{s\prime}}|X\right]=\frac{q}{\lambda^M}.$$

Case 2: $\phi^s < 1$ and $\gamma_b^s = 0$. In this case, I will compute (23)$\times(1-\phi^s)/q^B$+(24)$\times\frac{\lambda^M}{q}\phi^s$ and show that it trivially holds. The LHS is

$$(1-\phi^s)/q^B\times q^B+\frac{\lambda^M}{q}\phi^s\times\frac{q}{\lambda^M}=1.$$

The RHS is

$$\begin{aligned}&\mathbb{E}\left[\pi\frac{\frac{1-\phi^s}{q^B}}{\phi^s\frac{\lambda^M}{q}R^{i\prime}+(1-\phi^s)\frac{1}{q^B}}+(1-\pi)\frac{\frac{1-\phi^s}{q^B}}{\phi^s\frac{\lambda^M}{q}R^{s\prime}+(1-\phi^i)\frac{1}{q^B}}|X\right]\\&+\mathbb{E}\left[\pi\frac{\frac{\lambda^M}{q}\phi^sR^{i\prime}}{\phi^s\frac{\lambda^M}{q}R^{i\prime}+(1-\phi^s)\frac{1}{q^B}}+(1-\pi)\frac{\frac{\lambda^M}{q}\phi^sR^{s\prime}}{\phi^s\frac{\lambda^M}{q}R^{s\prime}+(1-\phi^s)\frac{1}{q^B}}|X\right]=1.\end{aligned}$$

Therefore, as long as (23) holds, equation (24) also holds. So I have shown that allocations that satisfy (14) also satisfy (23) and (24) .

D Proof of Proposition 4

In the steady state, the Euler equations of savers (23)-(24) become

$$1 = \pi \frac{\frac{1}{\bar{q}^B}}{\bar{\phi}^s \frac{\bar{\lambda}^M}{\bar{q}} \bar{R}^i + (1-\bar{\phi}^s)\frac{1}{\bar{q}^B}} + (1-\pi) \frac{\frac{1}{\bar{q}^B}}{\bar{\phi}^s \frac{\bar{\lambda}^M}{\bar{q}} \bar{R}^s + (1-\bar{\phi}^s)\frac{1}{q^B}},$$

$$1 = \pi \frac{\frac{\bar{\lambda}^M}{\bar{q}} \bar{R}^i}{\bar{\phi}^s \frac{\bar{\lambda}^M}{\bar{q}} \bar{R}^i + (1-\bar{\phi}^s)\frac{1}{\bar{q}^B}} + (1-\pi) \frac{\frac{\bar{\lambda}^M}{\bar{q}} \bar{R}^s}{\bar{\phi}^s \frac{\bar{\lambda}^M}{\bar{q}} \bar{R}^s + (1-\bar{\phi}^s)\frac{1}{q^B}}.$$

As $\bar{R}^i < \bar{R}^s$ and $0 < \bar{\phi}^s < 1$, it has to be $\frac{\bar{\lambda}^M}{\bar{q}} \bar{R}^i < \frac{1}{\bar{q}^B} < \frac{\bar{\lambda}^M}{\bar{q}} \bar{R}^s$. Therefore, $\frac{1}{\bar{q}^B} < \frac{\bar{\lambda}^M}{\bar{q}} \bar{R}^s < \bar{R}^s$.

One can show that

$$\begin{aligned}
\frac{\bar{\lambda}^M}{\bar{q}} \bar{R}^s &= \frac{\bar{\lambda}^M}{\bar{q}} \bar{r} + \bar{\lambda}^M F(\bar{\lambda}^M) + \int_{\bar{\lambda}^M}^{\infty} \lambda f_\phi(\lambda) d\lambda \\
&< \bar{r} + \bar{\lambda}^M F(\bar{\lambda}^M) + \int_{\bar{\lambda}^M}^{\infty} \lambda f_\phi(\lambda) d\lambda \\
&< \bar{r} + \bar{q} F(\bar{q}) + \int_{\bar{q}}^{\infty} \lambda f_\phi(\lambda) d\lambda \\
&= \bar{R}^i.
\end{aligned}$$

Therefore, $\frac{1}{\bar{q}^B} < \bar{R}^i$. In the steady state, the Euler equation of investors for government bonds (21) becomes

$$1 = \pi \frac{\frac{1}{\bar{q}^B}}{\bar{\phi}^i \bar{R}^i + (1-\phi^i)\frac{1}{\bar{q}^B}} + (1-\pi) \frac{\frac{1}{\bar{q}^B}}{\bar{\phi}^i \bar{R}^s + (1-\phi^i)\frac{1}{q^B}}.$$

Because $\frac{1}{\bar{q}^B} < \bar{R}^i$ and $\frac{1}{\bar{q}^B} < \bar{R}^s$, the inequality strictly holds, and $\bar{\phi}^i = 1$.

E Proof of Proposition 5

Consider a saver. It is straightforward to show that savers do not sell capital, i.e., $s^s = 0$, because the price of selling p is lower than the price of purchasing $p + \tau$. As a result, the saver's budget constraint can be expressed as

$$c^s + (p+\tau)k^{s\prime} + q^B b^{s\prime} = b + \left[r + (p+\tau)\bar{\lambda} + \eta^s\right] k.$$

Compare this constraint with the one in the asymmetric-information economy (12). If we set $p+\tau = \frac{q}{\lambda^M}$, and $\eta^s = F_\phi(\lambda^M)q - \frac{q}{\lambda^M}\int_0^{\lambda^M} \lambda f_\phi(\lambda)d\lambda$, this budget constraint coincide with (12).

Consider then an investor. As the price to purchase capital $p+\tau = \frac{q}{\lambda^M}$ is higher than the cost to invest (1), an investor will not purchase capital, i.e., $d^i = 0$. The investor's budget constraint can be expressed as

$$c^i + k^{i\prime} + q^B b^{i\prime} = b + \left(r + \bar{\lambda} + \eta^i\right) k + (p-1)s^i.$$

Compare this constraint with the one in the asymmetric-information economy (11). If $p = 1$ and $\eta^i(X) = F_\phi(q)q - \int_0^q \lambda f_\phi(\lambda)d\lambda$, the two equations are identical. Therefore, $\tau = (p+\tau)-1 = \frac{q}{\lambda^M} - 1$.

When individual investors' and savers' problems coincide with those in the asymmetric-information economy given q and λ^M, the aggregate variables $K^{i\prime}$, $K^{s\prime}$, $B^{i\prime}$, $B^{s\prime}$ are also the same as the asymmetric-information economy. The purchase of efficiency units by savers D^s_{SI} is the same as $\lambda^M D^s$. Given that $D^g_{SI} = \lambda^M D^g$, the sales of efficiency units by entrepreneurs is also the same as that in the asymmetric-information economy: $S^i = \pi K^p \int_0^q \lambda f_\phi(\lambda)d\lambda$.

It remains to verify that the tax and subsidy policies are revenue neutral and therefore the government budget constraint is unaltered.

$$\begin{aligned}
& \left[\pi\eta^i + (1-\pi)\eta^s\right] K^p \\
= \;& \pi \left[F(q)q - \int_0^q \lambda f_\phi(\lambda)d\lambda\right] K^p + (1-\pi)\left[F(\lambda^M)\lambda^M - \frac{q}{\lambda^M}\int_0^{\lambda^M} \lambda f_\phi(\lambda)d\lambda\right] K^p \\
= \;& \left[\pi F(q)q + (1-\pi)F(\lambda^M)\right] K^p - \left[\pi \int_0^q \lambda f_\phi(\lambda)d\lambda + (1-\pi)\frac{q}{\lambda^M}\int_0^{\lambda^M} \lambda f_\phi(\lambda)d\lambda\right] K^p \\
= \;& \frac{q}{\lambda^M}\left[\pi \int_0^q \lambda f_\phi(\lambda)d\lambda + (1-\pi)\int_0^{\lambda^M} \lambda f_\phi(\lambda)d\lambda\right] K^p \\
& - \left[\pi \int_0^q \lambda f_\phi(\lambda)d\lambda + (1-\pi)\frac{q}{\lambda^M}\int_0^{\lambda^M} \lambda f_\phi(\lambda)d\lambda\right] K^p \\
= \;& \left(\frac{q}{\lambda^M} - 1\right) S^i,
\end{aligned}$$

where the third equality uses the definition of λ^M in the asymmetric information economy. Therefore, I have shown that when $D^g_{SI} = \lambda^M D^g$ and $T_{SI} = T$, the government budget constraints are the same with and without asymmetric information.