

Optimal Fiscal and Monetary Policy with Collateral Constraints

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Abstract

We study the Ramsey optimal fiscal and monetary policy in an economy with financially constrained banks. Inflation reduces bank net worth and tightens their collateral constraint by revaluing their nominal assets and liabilities. The optimal policy balances tax distortions against costs of inflation on banks; thus, perfect tax smoothing is not optimal. Quantitatively, inflation plays a much smaller role in financing fiscal needs in the optimal policy compared to existing literature. With price stickiness and long-term government debt, optimal inflation is modest and persistent. The role of inflation in fiscal financing increases with the maturity of government debt.

Key words: Collateral constraints; Optimal fiscal and monetary policy; Bank net worth

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1 Introduction

How should fiscal and monetary policy be set in response to government spending shocks? Without defaulting on the outstanding debt, a government can either increase distortionary taxation or use inflation to reduce the real value of government debt denominated in domestic currency. As deficits and public debt are approaching historical highs in major economies, many economists have argued for a higher inflation target in order to alleviate the public debt burden.¹

Standard models in the optimal policy literature prescribe the use of state-contingent inflation to smooth tax distortions over time and across states (Lucas and Stokey, 1983; Chari et al., 1991). In periods of higher government consumption, higher inflation reduces the real value of nominal debt and attenuates the increase in taxes. Such policy is optimal, because inflation is a lump-sum tax on government debt holders from the ex post point of view.

This paper considers a novel cost of inflation on commercial banks. Higher inflation reduces commercial bank net worth by revaluing nominal fixed-income claims. First, commercial banks with balance sheet exposure to government debt directly suffer losses when the government monetizes its debt.² Second, due to the maturity mismatch between bank assets and liabilities, a persistent increase in the inflation rate (e.g., a higher inflation target) causes bank asset values to drop faster than liability values. Commercial banks play an important role in financial intermediation; losses borne by financially constrained banks hamper the credit supply and dampen real economic activity.

In this paper, we show that introducing the cost of inflation to commercial banks greatly

¹For example, see Blanchard et al. (2010), Krugman (2013), and Rogoff (2008). Of course, reducing the public debt burden is not the only reason that higher inflation is desirable. Higher inflation is also advocated to address wage rigidity, reduce household debt, and reduce the real interest rate when the nominal interest reaches its zero lower bound.

²For instance, the holdings of Japanese government bonds by Japan's banks equate to 900% of their Tier 1 capital (Jenkins and Nakamoto, 2012). When the Bank of Japan started qualitative and quantitative monetary easing in 2013 with the goal of reaching the 2% inflation target, fears arose that banks would bear large losses if the inflation rate were raised.

changes the prescription of optimal fiscal and monetary policy, as the response of inflation to higher government spending becomes quantitatively much smaller than in standard models. Thus, our analysis shows how the interaction between financial friction and the bank balance sheet costs of inflation provides a novel justification for the undesirability of government debt monetization.

Our model builds on [Angeletos et al. \(2013\)](#). The economy is populated with a large number of bankers who provide funds to firms. Firms are subject to idiosyncratic productivity shocks. For a high-productivity firm to acquire more productive resources, its banker needs to raise external funds through collateralized borrowing. Bankers hold nominal government debt and physical capital, both of which serve as collateral. The government finances fiscal expenditures and interest payments by imposing distortionary labor taxes and using state-contingent inflation.

We study the response of Ramsey optimal fiscal and monetary policy to fiscal shocks. When the government generates inflation to reduce the real value of debt, bankers' collateral constraints are tightened, which impedes resource reallocation across heterogeneous firms and distorts their investment decisions. Therefore, inflation is no longer a lump-sum tax even ex post, and perfect smoothing of the labor tax is no longer optimal. The optimal policy response to shocks features a combination of a higher tax rate and a higher inflation rate, as the government should balance the cost of inflation with the cost of distortionary taxes.

We calibrate the model to the U.S. economy and perform a decomposition analysis to quantify the financing of the innovations in government spending. It is optimal for countries like the U.S. to finance 55.65% of the increase in fiscal needs through inflation and address the remainder fiscal needs through higher tax revenues. In contrast, in standard models without the collateral constraints of bankers, inflation finances all increases in government expenditures, and tax revenues are left constant. Our model also implies that the volatility of inflation is only half the size as in the standard model. To the extent that inflation volatility

in the standard model is extreme and at odds with the data (Chari et al., 1991), our model provides a rationale for small inflation volatility in the optimal policy design.

Price stickiness is another reason levied against the use of inflation in the optimal policy (Schmitt-Grohé and Uribe, 2004; Siu, 2004). However, when the government issues long-term debt, large changes in the value of the debt can be produced by modest and persistent inflation. Therefore, inflation still plays an important role in fiscal financing (Leeper and Zhou, 2013; Sims, 2013). We extend our benchmark model to incorporate price stickiness and long maturity of government debt. With the degree of price stickiness and the maturity of debt calibrated to the U.S. economy, it is optimal for the government to finance 31.38% of the higher fiscal spending through inflation. Consistent with the data, the optimal response of inflation is modest and persistent. 25.95% of the increase in fiscal spendings is financed by inflation in future periods, compared to 5.43% financed by inflation in the initial period when the fiscal shock arrives. The maturity of government debt has a great impact on policy prescriptions. For instance, when the debt matures in one period (quarter), inflation only finances 13.60% of higher government spending in the optimal policy.

As a case study, we analyze the model implication for wartime financing in the wars in Iraq and Afghanistan. In the full model with bank balance sheet costs, price stickiness, and long-term debt, the average increase in annual inflation rate is 0.44%, accompanied by an average increase in the labor tax rate of 1.08 percentage points.

Related Literature. This paper relates to the aforementioned literature on optimal fiscal and monetary policy using a Ramsey approach in both flexible and sticky price settings. Our model builds on Angeletos et al. (2013), which considers optimal fiscal policy when real government debt serves as collateral and focuses on the determination of a long-run debt level. We introduce nominal government debt and argue that collateral constraints are also important in the design of monetary policy.

The mechanism of this model relates to the vast theoretical and empirical literature on the liquidity role of public debt (e.g., Woodford, 1990; Holmstrom and Tirole, 1998;

Aiyagari and McGrattan, 1998; Krishnamurthy, 2002). In this model, bankers choose to hold government debt on their balance sheet, because it provides liquidity in the capital reallocation process. Inflation reduces the real value of debt and hampers its liquidity role.

This paper also contributes to the literature that examines the redistribution effect of inflation by revaluing nominal contracts in general equilibrium models. Gomes et al. (2016) models the effect of unanticipated inflation on the real value of nominal corporate debt and the severity of debt overhang. A number of works study the redistribution effect of inflation on nominal household debt (Auclert, 2016; Garriga et al., 2013; Meh et al., 2010). This literature shares the view that nominal contracts create a link between inflation and the real economy and serve as an important source of monetary non-neutrality, even with fully flexible prices. Our work contributes to the literature by highlighting the importance of the nominal positions of the banking sector.

At a conceptual level, this paper also relates to a growing literature on the link between sovereign default and bank fragility (Gennaioli et al., 2014; Sosa-Padilla, 2012; Bolton and Jeanne, 2011; Guerrieri et al., 2013). As inflation can be viewed as a partial default on government liabilities, our model shares with this literature the idea that the repudiation of government debt tightens financial constraints on the banking sector. This literature usually assumes a lack of commitment on the part of the government. We instead study optimal policy under full government commitment and focus exclusively on financial frictions.

Empirical Relevance. How large is the effect of inflation on the real value of the assets, liabilities, and net worth of U.S. commercial banks? In Cao (2016), we quantify this effect using bank-level data from the Bank Reports of Conditions and Income (call reports) filed quarterly by U.S. commercial banks. We first document that the average maturity of nominal assets is longer than nominal liabilities by about five years. In the spirit of Doepke and Schneider (2006), we then consider a hypothetical scenario of a 1% unanticipated and permanent increase in the inflation rate that causes a parallel shift in the yield curve, and we study its revaluing effect on bank balance sheets. We find an average 15% loss of Tier 1

capital for U.S. commercial banks in this scenario. The amount of loss is similar for banks that do not hold interest rate derivatives and therefore do not hedge interest rate risk.³ Therefore, even a moderate inflation episode reduces bank net worth substantially through the revaluation of bank nominal assets and liabilities.⁴

Roadmap. The rest of the paper is organized as follows. We describe the benchmark model and the Ramsey optimal policy problem in section 2 and explore the quantitative results in section 3. In section 4, we extend the model to incorporate price stickiness and the long maturity of government debt. We conclude in section 5.

2 Model

2.1 Environment

The economy consists of a continuum of identical households. Within each household reside equal masses of bankers $i \in [0, 1]$ and workers $j \in [0, 1]$. Members in each household share consumption perfectly. Each worker supplies labor in a competitive labor market and earns a wage income. Each banker channels funds to a firm that produces final goods. We ignore financial frictions between a banker and their firm; thus, each banker effectively owns the firm.⁵ We use i to index the firm owned by banker i .

Preference and Technology. Preferences over stochastic processes for the household con-

³Recent empirical studies (Begenau et al., 2015; Gomez et al., 2016) find that holdings of interest rate derivatives at best partially hedge banks' exposure to interest rate risk and inflation risk. In particular, Begenau et al. (2015) show that net derivative positions tended to amplify, not offset, balance sheet exposure to interest rate risk for the four largest U.S. banks from 1997 to 2004.

⁴Our results are comparable to that in Bank of Japan (2013), who performs a similar analysis of Japanese commercial banks. They find that a 1% parallel shift in the yield curve causes an average 20% loss of Tier 1 capital for Japanese commercial banks in 2012.

⁵As each banker is the owner of a firm, in the text below, we use banker i and firm i interchangeably, with a slight abuse of notation. The assumption of no friction between bankers and firms is similar to Gertler and Kiyotaki (2010). It allows us to focus on the bankers' balance sheets and how their borrowing capacity is limited by their net worth. As in Gertler and Kiyotaki (2010), our analysis focuses on the financial constraints of banks (lenders) rather than borrowers (e.g., Bernanke et al., 1999; Iacoviello and Minetti, 2006).

sumption $\{c_t\}_{t \geq 0}$ and labor supply $\{h_{j,t}\}_{\geq 0}$ of each worker j are ordered by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\rho} - 1}{1-\rho} - \chi \frac{\int_0^1 h_{j,t}^{1+\epsilon} dj}{1+\epsilon} \right). \quad (1)$$

Firm i uses $k_{i,t}$ units of physical capital and $n_{i,t}$ units of labor to produce output $y_{i,t}$:

$$y_{i,t} = z_{i,t} F(k_{i,t}, n_{i,t}),$$

where F has decreasing returns to scale, with $F(k, n) = k^\alpha n^\theta$ and $\alpha + \theta < 1$. $z_{i,t}$ is an idiosyncratic productivity shock independent and identically distributed across both bankers and time. $z_{i,t}$ can take two values:

$$z_{i,t} = \begin{cases} z^H & \text{with probability } \sigma \\ z^L & \text{with probability } 1 - \sigma. \end{cases}$$

Idiosyncratic shocks generate a need for capital reallocation.

Physical capital depreciates at rate δ . Aggregate capital stock a_t is the sum of the stock of undepreciated capital and current investment i_t :

$$a_t = (1 - \delta)a_{t-1} + i_t.$$

Aggregate Uncertainty. The only source of aggregate uncertainty in this model is a stochastic government consumption g_t . Aggregate history up until time t is $g^t = (g_0, \dots, g_t)$, and the time-0 probability of g^t is denoted by $\Pr(g^t)$. To save on notation, we use X_t to denote a random variable that is a function of the history g^t .

Aggregate output y_t is divided between household consumption, investment expenditures, and government consumption:

$$c_t + a_t + g_t = (1 - \delta)a_{t-1} + y_t. \quad (2)$$

Government Policy. The government consists of fiscal and monetary authorities. The fiscal authority imposes proportional taxes on labor income with a tax rate τ_t and issues one-period nominal bonds B_t . The monetary authority decides upon the nominal interest rate R_t^B paid on B_t . The following consolidated government budget constraint must hold:

$$\tau_t w_t h_t + \frac{B_t}{P_t} = \frac{R_{t-1}^B B_{t-1}}{P_t} + g_t. \quad (3)$$

2.2 Capital market and collateral constraint

We describe the sequence of activities within each time period t in Figure 1. At the beginning of period t , workers and bankers separate, and they cannot meet each other until the end of the period. We assume that before the separation, each household shares all the assets accumulated during the previous period among all the bankers in the household. Therefore, each banker i holds an equal share of the household's assets, which consist of physical capital a_{t-1} and government bonds B_{t-1} .

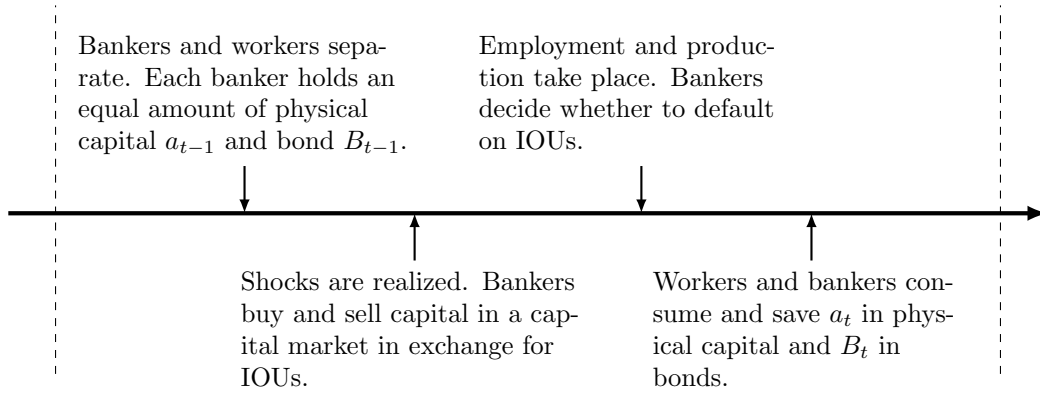


Figure 1: Timeline of activities within period t .

After the idiosyncratic and aggregate shocks realize, high-productivity bankers want to scale up their production and use more capital in production than the amount they have ($k_{i,t} > a_{t-1}$ if $z_{i,t} = z^H$). They can buy the amount $k_{i,t} - a_{t-1}$ from other bankers in a competitive capital market at price q_t units of consumption goods. A buyer of capital does not pay for the capital until production is finished; therefore, at this stage, he or she issues

private IOUs to the seller.

However, after employment and production take place, buyers could repudiate their IOUs. In this case, sellers could confiscate some fraction of a buyer i 's assets, which is ξ fraction of capital installed in the firm $k_{i,t}$ and the total real payoff from government debt holding $R_{t-1}^B B_{t-1}/P_t$.⁶ Therefore, a buyer faces the incentive constraint that the total value of IOUs he or she issues cannot exceed the total value of the confiscable assets; that is:

$$q_t (k_{i,t} - a_{t-1}) \leq \xi k_{i,t} + \frac{R_{t-1}^B B_{t-1}}{P_t}.$$

Rearranging this inequality constraint yields:

$$k_{i,t} \leq \underbrace{\frac{1}{q_t - \xi}}_{\text{leverage}} \times \underbrace{\left(q_t a_{t-1} + \frac{R_{t-1}^B B_{t-1}}{P_t} \right)}_{\text{net worth}}. \quad (4)$$

This constraint implies that $k_{i,t}$ is limited by the banker's total net worth. $\frac{1}{q_t - \xi}$ is the leverage ratio: for each unit of capital that banker i uses in production, he could credibly pledge ξ fraction, and therefore he needs to secure the remaining $q_t - \xi$ fraction using his own net worth.

The model broadly captures the mismatch of maturity of bank balance sheets observed in the data and the effect of inflation. In this model, part of the bankers' assets are nominal government bonds that mature in one period. Bankers' liabilities are only within-period. Thus, inflation reduces the real value of bank assets but not the liabilities, reducing the net worth of bankers and tightening their collateral constraints.

Remark. We have assumed that bankers in the same household reshuffle assets among themselves at the end of each period. This assumption allows us to study heterogeneity and

⁶Due to spatial separation, bankers and workers cannot reshuffle the resources among themselves after shocks realize. For the same reason, a banker cannot pledge the wage incomes of workers in the same household as collateral. A banker cannot credibly pledge his or her own future income either. This assumption that human capital is inalienable has been followed in much of the literature on financial frictions since [Hart and Moore \(1994\)](#). As a buyer's default happens after production when physical capital can be converted to consumption goods one for one, the real price of capital $k_{i,t}$ at this point is 1.

capital reallocation while maintaining the structure of a representative household. Absent this assumption, we would need to keep track of the distribution of assets across bankers. This would greatly increase the computational burden for the Ramsey optimal policy.

2.3 Households' decision problem and competitive equilibrium

Production decisions only depend on the current productivity shock $z_{i,t}$, because all bankers have the same asset holdings before shocks realize. Therefore, we denote variables regarding production decisions by superscript s , where $s = L$ if $z_{i,t} = z^L$, and $s = H$ if $z_{i,t} = z^H$. As workers are identical, they all work the same amount, and $h_{j,t} = h_t$ for all j .

Since all household members share their consumption risks, a household faces a consolidated end-of-period budget constraint:

$$c_t + a_t + \frac{B_t}{P_t} = [\sigma v_t^H + (1 - \sigma)v_t^L] + (1 - \tau_t) w_t h_t + \frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1}. \quad (5)$$

A household's income consists of bankers' profits, workers' after-tax labor income, and household savings income. Each banker earns a profit from his or her firm:

$$v_t^s = z^s F(k_t^s, n_t^s) - w_t n_t^s - [q_t - (1 - \delta)] k_t^s. \quad (6)$$

A household's decision problem is to choose $\{k_t^s, n_t^s, h_t, c_t, a_t, B_t\}_{t \geq 0}$ to maximize utility (1), subject to the end-of period budget constraint (5) and the collateral constraint (4). The workers' labor supply decision satisfies:

$$(1 - \tau_t) w_t = \chi \frac{h_t^\epsilon}{c_t^{1-\rho}}. \quad (7)$$

The labor and capital demand conditions of a type- s bank are:

$$z^s F_n(k_t^s, n_t^s) = w_t, \quad (8)$$

$$z^s F_k(k_t^s, n_t^s) = q_t - (1 - \delta) + \mu_t^s, \quad (9)$$

where $\mu_t^s U_{c,t}$ is the multiplier on the collateral constraint. In equilibrium, the collateral constraint never binds for low-productivity bankers, and $\mu_t^L = 0$, because they sell capital ($k_t^L < a_{t-1}$). Capital price q_t is determined by the gross return to capital of the unconstrained low-productivity bankers, which is the opportunity cost to sell capital. As there is no friction in the labor market, the marginal product of labor equalizes across the two types of bankers.

Lemma 1 characterizes the aggregate economy, given the allocation of capital between the two types of bankers denoted by $x_t \equiv \frac{k_t^H}{a_{t-1}}$.

Lemma 1 (*The aggregate economy*)

1. *Aggregate output satisfies:*

$$y_t = \Gamma(x_t) a_{t-1}^\alpha h_t^\theta, \quad (10)$$

where

$$\Gamma(x) = \left[\sigma z^{H \frac{1}{1-\theta}} x^{\frac{\alpha}{1-\theta}} + (1 - \sigma) z^{L \frac{1}{1-\theta}} \left(\frac{1 - \sigma x}{1 - \sigma} \right)^{\frac{\alpha}{1-\theta}} \right]^{1-\theta}.$$

$\Gamma(x)$ is maximized at an interior point:

$$x^* \equiv \arg \max_x \Gamma(x) = \frac{z^{H \frac{1}{1-\alpha-\theta}}}{\sigma z^{H \frac{1}{1-\alpha-\theta}} + (1 - \sigma) z^{L \frac{1}{1-\alpha-\theta}}}.$$

2. *Equilibrium wage rate equals the aggregate marginal product of labor:*

$$w_t = \theta \Gamma(x_t) a_{t-1}^\alpha h_t^{\theta-1}. \quad (11)$$

3. *Equilibrium capital price deviates from the gross return of capital if and only if high-*

productivity bankers' collateral constraint strictly binds:

$$q_t = 1 - \delta + \underbrace{\alpha \Gamma(x_t) a_{t-1}^{\alpha-1} h_t^\theta}_{\text{aggregate MPK}} \underbrace{\left[z^L \left(\frac{1 - \sigma x_t}{1 - \sigma} \right)^{\alpha+\theta-1} \Gamma(x_t)^{-1} \right]^{\frac{1}{1-\theta}}}_{\text{deviation from aggregate MPK}} \equiv \mathbf{q}(a_{t-1}, h_t, x_t). \quad (12)$$

In other words, the term in the bracket equals 1 if and only if $x_t = x^*$. Otherwise, it is less than 1.

4. The multiplier on high-productivity bankers' collateral constraint is:

$$\mu_t^H = \frac{1}{\sigma} \Gamma'(x_t) a_{t-1}^{\alpha-1} h_t^\theta \equiv \boldsymbol{\mu}^H(a_{t-1}, h_t, x_t). \quad (13)$$

$\mu_t^H = 0$ if and only if $x_t = x^*$.

Since the production technology has decreasing returns to scale, there exists an efficient level x^* in the absence of collateral constraints. When the high-productivity bankers' constraint binds, capital allocations are suboptimal ($x_t < x^*$). Therefore, the endogenous total factor productivity (TFP) falls below the efficient level $\Gamma(x^*)$, and the capital price is below what is implied by the aggregate gross return of capital. Wage rate always equals the aggregate marginal product of labor, because the labor market is frictionless.

The existence of the collateral constraint also introduces a wedge on the inter-temporal margin, which can be illustrated by the Euler equations of the household:

$$U_{c,t} = \beta \mathbb{E}_t U_{c,t+1} q_{t+1} \left(1 + \frac{\sigma \mu_{t+1}^H}{q_{t+1} - \xi} \right), \quad (14)$$

$$U_{c,t} = \beta \mathbb{E}_t U_{c,t+1} \frac{R_t^B}{P_{t+1}} \left(1 + \frac{\sigma \mu_{t+1}^H}{q_{t+1} - \xi} \right). \quad (15)$$

If the collateral constraint strictly binds with positive probability in period $t + 1$, the associated Lagrange multiplier introduces a wedge between the rate of return of capital (or government bonds) and the inter-temporal marginal rate of substitution, which distorts the household's investment decision. At the same time, government bonds are priced at a

premium relative to an asset that is an equally good form of saving but cannot serve as collateral.⁷ This lowers the debt serving costs and allows the government to reduce taxes.

A *competitive equilibrium* is defined in the usual way. The household (workers and bankers) solves its problems, taking prices as given. The wage rate, capital price, and bond interest rate clear the labor, capital, and bond markets. In addition, the government budget constraint is satisfied. Equilibrium can be summarized by a set of allocations $\{y_t, a_t, h_t, c_t, x_t, B_t\}_{t \geq 0}$, prices $\{q_t, w_t, P_t, \mu_t^H\}_{t \geq 0}$, and fiscal and monetary policies $\{\tau_t, R_t^B\}_{t \geq 0}$ satisfying (3)–(5), (7), (10)–(15), $\mu_t^H \geq 0$, and the complementary slackness condition⁸, given initial household asset positions a_{-1} and $R_{-1}^B B_{-1}$ and the process of government consumption shocks $\{g_t\}_{t \geq 0}$.

2.4 Ramsey optimal policy

The Ramsey optimal fiscal and monetary policy is the process $\{\tau_t, R_t^B\}_{t \geq 0}$ associated with the competitive equilibrium that yields the highest social welfare:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\rho}}{1-\rho} - \chi \frac{h_t^{1+\epsilon}}{1+\epsilon} \right) \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t).$$

We follow the primal approach in the Ramsey policy literature, which substitutes for prices and policy instruments so that the Ramsey planner directly chooses real allocations. Lemma 1 expresses $\{w_t, q_t, \mu_t^H\}$ as functions of $\{a_{t-1}, h_t, x_t\}$. The labor tax rate is the wedge between the marginal rate of substitution and the marginal product of labor:

$$\tau_t = 1 + \frac{U_{h,t}}{U_{c,t}} \frac{1}{\Gamma(x_t) F_h(a_{t-1}, h_t)}. \quad (16)$$

⁷This is consistent with the observations that government bonds pay a lower return due to liquidity attributes (Krishnamurthy and Vissing-Jorgensen, 2012) and that the “natural rate of interest” declines as credit tightens (Eggertsson and Krugman, 2012).

⁸The complementary slackness condition is:

$$\mu_t^H \left[\frac{1}{q_t - \xi} \left(q_t a_{t-1} + \frac{R_{t-1}^B B_{t-1}}{P_t} \right) - x_t a_{t-1} \right] = 0.$$

We substitute for the real holding-period return on government debt $r_t^b = \frac{R_t^B P_{t-1}}{P_t}$ and derive the flow implementability constraint:

$$\beta \mathbb{E}_{t-1} [U_{c,t} c_t + U_{h,t} h_t - U_{c,t} (1 - \alpha - \theta) y_t] + \beta \mathbb{E}_{t-1} U_{c,t} (a_t + b_t) = U_{c,t-1} (a_{t-1} + b_{t-1}), \quad (17)$$

where $b_t \equiv \frac{B_t}{P_t}$ is the real government debt. We also combine the collateral constraint with the government budget constraint to substitute for r_t^b :

$$x_t a_{t-1} (q_t - \xi) \leq q_t a_{t-1} + \left(\theta y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - g_t \right). \quad (18)$$

We now establish the equivalence between the primal approach and the original Ramsey problem. The proof is in Appendix [A.2](#).

Lemma 2 (*Primal approach*) *An aggregate allocation $\{a_t, h_t, x_t, c_t, b_t\}_{t \geq 0}$ can be supported as a competitive equilibrium under appropriately chosen $\{r_t^b, \tau_t\}_{t \geq 0}$ if and only if it satisfies the social resource constraint (2), the flow implementability constraint (17), the Euler equation (14), the collateral constraint (18), $\mu^H(\cdot) \geq 0$, and the household complementary slackness condition. Price functions $\mathbf{q}(\cdot)$ and $\mu^H(\cdot)$ are defined in equations (12) and (13).*

The Ramsey problem determines the real state-contingent return on debt r_t^b . As the government adjusts the real return through state-contingent inflation, the Ramsey problem determines the state-contingent component of inflation. However, the expected inflation rate is not determined in the Ramsey problem. Without loss of generality, we assume zero expected inflation.⁹

⁹Formally, denote the gross inflation rate by π_t , then the state-contingent component of inflation is defined by:

$$\pi_t \mathbb{E}_{t-1} \frac{1}{\pi_t} = \frac{\mathbb{E}_{t-1} r_t^b}{r_t^b}.$$

We impose the assumption that $\mathbb{E}_{t-1} \frac{1}{\pi_t} = 1$. The result of zero inflation can emerge from a sticky-price version of our model (see section 4) with a tiny amount of price stickiness. In the literature, expected inflation can be determined either by incorporating price stickiness, which drives the expected inflation rate to 0, or by introducing a non-interest-bearing government liability (money stock) that leads to the Friedman rule (e.g., [Chari et al., 1991](#)). Both features are absent in our flexible-price model.

In the rest of the paper, we compare the Ramsey optimal policy with that of an otherwise identical model but without the bank collateral constraint (we refer to it as the “frictionless model” below). In this alternative economy, the allocation of capital is always optimal (i.e., $x_t = x^*$). Lemma 3 characterizes the primal approach of the Ramsey problem in the frictionless model. Proof is in Appendix A.3.

Lemma 3 (*Primal approach in the frictionless model*) *Allocations $\{a_t, h_t, c_t, b_t\}_{t \geq 0}$ can be supported as a competitive equilibrium in an economy without the collateral constraint under approximately chosen $\{r_t^b, \tau_t\}_{t \geq 0}$ if and only if the social resource constraint (2), the flow implementability constraint (17), and the Euler equation (14) hold with $x_t = x^*$.*

Due to the lack of previous commitment from the government, the time-0 Ramsey policy differs from that of $t \geq 1$. In the remainder of the paper, we focus on the continuation problem of $t \geq 1$ where the government can fully commit to its policy. Abstracting away from the government commitment issue allows us to focus exclusively on how the collateral constraint affects the optimal policy design. Therefore, it provides a clean benchmark.

2.5 A quasi-linear example

In this section, we assume that the household’s utility function is linear in consumption ($\rho = 0$):

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(c_t - \chi \frac{h_t^{1+\epsilon}}{1+\epsilon} \right).$$

Under this assumption, Proposition 1 establishes that perfect tax smoothing emerges in the optimal policy in the frictionless model. The proof is in Appendix A.4.

Proposition 1 (*Ramsey policy in the frictionless model.*) *In the absence of the collateral constraint, the Ramsey problem features a constant tax rate and productions across dates and states:*

$$a(g^t) = a^*, \quad \text{for } t \geq 0 \text{ and } \forall g^t,$$

$$h(g^t) = h^*, \quad y(g^t) = y^*, \quad \tau(g^t) = \tau^*, \quad \text{for } t \geq 1 \text{ and } \forall g^t,$$

where a^* , h^* , y^* and τ^* are constants independent of history g^t .

Under the assumption of quasi-linear preference, the optimal labor tax rate is exactly constant in the absence of collateral constraints.¹⁰ As the government consumption fluctuates, the inflation rate and the real return on government debt (r^b) fluctuate to satisfy the government budget constraint, and the labor tax rate is maintained constant regardless of the realization of government consumption shock. Intuitively, higher inflation and lower real return on debt resemble a lump-sum tax on households' wealth ex post, but a proportional labor tax is distortionary, and the efficiency loss is convex.

We use a simple numerical example to illustrate the optimal policy in the model with collateral constraints and compare it with the frictionless model. The numerical example is set in annual frequency. We assume that government consumption shock follows a two-state Markov process with a symmetric transition matrix and the high state g^H is about 10% higher than the low state g^L . We choose the transition probability such that each state has an average duration of 15 years.¹¹ In Figure 2, we start the economy at a level of government debt to which the economy converges after a long sequence of g^L . We choose ξ such that the debt-to-GDP ratio converges to 61%, which is the value for the U.S. before the 2008 crises (2007Q3).¹² We set the initial debt-to-GDP ratio in the frictionless economy to be the same as the model with constraints.

As shown in Figure 2, in year 11, the government consumption switches from g^L to g^H and

¹⁰Our results generalize those of Chari et al. (1991) by incorporating physical capital into the model. On the other hand, it can be shown that in their model, optimal labor tax is constant whenever utility is separable in consumption and leisure, but this is not true in our model. Proposition 1 also shows that after the initial period, capital $a(g^t)$, labor $h(g^t)$, and output $y(g^t)$ are independent of history and state. This is because government consumption shock is the only aggregate shock in this model. Capital, labor, and output will fluctuate if, for example, an aggregate productivity shock is introduced.

¹¹This process is in line with the government consumption process for the post-war U.S. The U.S. data in the sample period 1949Q1–2007Q4 show that annual government consumption averaged about 15.1% of GDP, with a standard deviation of 1.75% and an autocorrelation of 0.60. The distribution is also very symmetric.

¹²Other parameters values are: $\beta = 0.96$, $\chi = 3.4$, $\epsilon = 1$, $\alpha = 0.283$, $\theta = 0.566$, $\delta = 0.1$, $\sigma = 0.5$, $z^H = 1.822$ and $z^L = 1$.

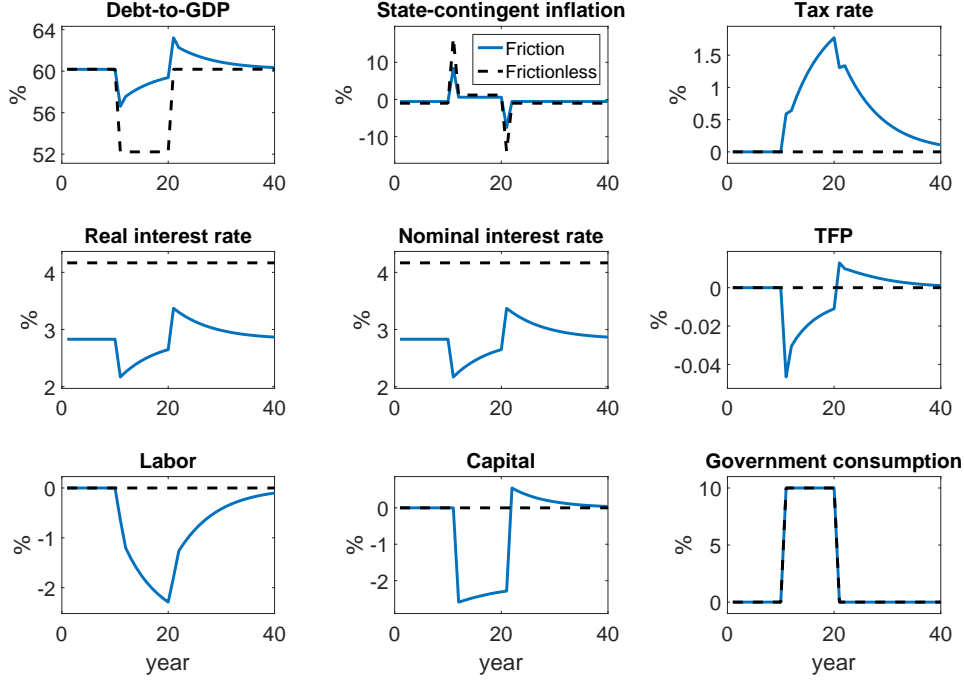


Figure 2: Stochastic stimulations.

Note: High government consumption shock g^H occurs in year 11 and lasts for 10 years. The debt-to-GDP ratio, the inflation rate, and the interest rates are measured in percentage points; the tax rate is measured in the deviations from its value in year 10, and other variables are measured in percentage deviations from their values in year 10.

lasts for 10 years. Consistent with Proposition 1, the frictionless economy features perfect tax smoothing. The government generates state-contingent inflation sufficient to reduce the real debt by exactly the sum of expected increases in the current and future government expenditure. As long as the g^H state continues, real debt and the debt-to-GDP ratio remain at the lower level.

In the economy with collateral constraints, perfect tax smoothing through monetizing outstanding debt is no longer optimal. As the economy switches from g^L to g^H , the optimal policy features a combination of higher inflation and higher tax rate. Inflation reduces the real value of government debt, and the resultant dearth of collateral hinders capital reallocation and dampens TFP. As government consumption shocks are persistent, the government continues to issue less debt in the subsequent periods as g^H continues. Therefore, the economy endures a persistent lack of collateral and low efficiency of capital allocation,

which reduces the incentive to invest. On the other hand, inflation allows the government to reduce the size of tax increases. In addition, the dearth of collateral raises the liquidity premium and lowers the real interest rate paid on debt.¹³

3 Quantitative analysis

We now relax the quasi-linear preference assumption and explore the quantitative property of the model. We adopt a log-linear approximation around the non-stochastic steady state where the collateral constraint strictly binds. We assume that the collateral constraint always binds when solving the model, and we later verify that this is the case for the size of shock we consider.¹⁴

3.1 Calibration

Table 1 summarizes the parameters. The model is computed at a quarterly frequency, and the discount factor β is set to 0.99. We set $\epsilon = 1$, implying a Frisch elasticity of labor supply of 1. This number, in line with the recommendation of Chetty et al. (2011), is appropriate given that our model does not distinguish between intensive and extensive margins of employment.

Regarding the production technology, the overall returns to scale $\alpha + \theta$ are set to 0.85, and the share of labor θ is set to two thirds of 0.85 (Midrigan and Xu, 2014; Basu and Fernald, 1997). We assume a symmetric idiosyncratic productivity shock process by setting $\sigma = 0.5$ and normalize the low realization of productivity z^L to 1. We choose the high realization z^H such that the standard deviation of the logarithm idiosyncratic productivity is 0.3. This value is consistent with the estimated size of TFP innovations using U.S. manufacturing

¹³Due to the assumption that the expected inflation rate is zero, the nominal interest rate equals the real interest rate in our model. As shown in the fifth panel, the interest rate decline is not large enough to hit the zero lower bound.

¹⁴In Appendix A.5, we evaluate the accuracy of linearized solutions by showing that 1) the linearized solution and the global solution for the model with the quasi-linear utility function are very similar and 2) the linearized and quadratic solutions for the model with the general utility function are very similar.

	Parameters	Value	Target/Source
<i>Preferences</i>			
Household discount factor	β	0.990	
Relative risk aversion	ρ	2.000	
Disutility of labor	χ	3.400	Gertler and Karadi (2011)
Inverse Frisch elasticity	ϵ	1.000	Chetty et al. (2011)
<i>Production Technology</i>			
Capital share of output	α	0.283	one third of 0.850
Labor share of output	θ	0.566	two thirds of 0.850
Depreciation rate of capital	δ	0.025	
Probability of z^H	σ	0.500	
High idiosyncratic productivity	z^H	1.822	STD of $\log(z_{i,t})$ is 0.3
Low idiosyncratic productivity	z^L	1.000	normalized
<i>Financial Friction</i>			
Pledgeable share of capital	ξ	0.340	steady-state debt-to-GDP ratio is 61%
<i>Government Consumption</i>			
SS government consumption	\bar{g}/\bar{y}	0.159	estimated
persistence of g shock	ρ^g	0.890	estimated
std of g shock	σ^g	1.53%	estimated

Table 1: Parameters

firms (Asker et al., 2013).¹⁵ We estimate a first-order autoregressive process for aggregate government consumption from 1948Q1–2007Q3. The standard deviation is 1.53%, and the autocorrelation is 0.89.

The parameter ξ dictates the tightness of the collateral constraint, so it affects the amount of debt the government issues in the optimal policy. We calibrate ξ such that the steady-state debt-to-GDP ratio in the optimal policy matches the data (61% in 2007Q3). In our model, the size of government debt captures both the tax-saving benefit of inflation and the cost of inflation to banks, as government debt is the only nominal asset of banks.¹⁶ Our parameter choice implies that a 5% cumulative inflation causes 1.15% bank loss of net worth, while it is 3.7% in the data as documented in Cao (2016). Thus, the model understates the cost of

¹⁵As shown in Asker et al. (2013), the firm-level productivity is persistent, with an autocorrelation coefficient of 0.8. Therefore, the standard deviation of the productivity is $0.3/\sqrt{1-0.8^2} = 0.5$. In our model, idiosyncratic productivity is i.i.d. We perform a conservative calibration by calibrating the standard deviation to the size of productivity innovations rather than the productivity process in the data.

¹⁶In our model, government debt is the only nominal asset on bank balance sheets. In Cao (2015), we extend our model to allow banks to hold both nominal loans to firms and nominal government debts. State-contingent inflation thus affects bank net worth through both types of nominal assets. We revisit the Ramsey optimal fiscal and monetary policy in that richer environment, and the quantitative results are similar to the current model.

inflation to the bank net worth. In section 3.4, we study the robustness of the results to changes in ξ .

In the literature on financial frictions, financial friction parameters are usually calibrated to match either the yield or the yield spread of assets (Gertler and Karadi, 2011; Gertler and Kiyotaki, 2010; Del Negro et al., 2016). Our choice of ξ implies that the steady-state liquidity premium in our model is 0.64%.¹⁷ It is broadly in line with the estimation in Krishnamurthy and Vissing-Jorgensen (2012) that the average liquidity premium from 1926–2008 is 0.46%.

3.2 Results

Fiscal financing decomposition. We do the following decomposition to account for the sources of fiscal financing after an innovation to government consumption. Using the first-order-approximation of the inter-temporal government budget constraint, the present value of increases in government consumption is financed by the present value of increases in tax revenue and state-contingent inflation and the present value of the decreases in the real interest rate. A higher real interest rate is a negative news for fiscal financing, as future primary surpluses are now discounted at a higher rate:

$$\underbrace{\sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{g}}{\bar{b}} \tilde{g}_s}_{\text{government consumption}} = \underbrace{\sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{T}}{\bar{b}} \tilde{T}_s}_{\text{tax revenue}} + \underbrace{\frac{1}{\bar{\pi}} \hat{\pi}_t}_{\text{inflation}} - \underbrace{\sum_{s=t+1}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \hat{r}_s^b}_{\text{real interest rate}}. \quad (19)$$

Here we use \bar{X} , \hat{X} , and \tilde{X} to denote the steady-state level, the level deviation from the steady state, and the percentage deviation from the steady state of variable X , respectively. See Appendix A.6.1 for derivations.

Table 2 shows decompositions for both the frictionless economy and the economy with collateral constraints. In the frictionless economy, inflation finances more than 100% of the present value of the increase in government consumption. This is because of the negative

¹⁷The liquidity premium is defined as $4(1/\beta - \bar{r}^b)$. It is the difference between interest rates on government debt and an otherwise identical asset with no collateral value.

	Frictionless model	Friction model
Tax revenue	-2.25%	51.87%
State-contingent inflation	114.68%	55.65%
Real interest rate	-12.43%	-7.52%

Table 2: Decomposition of fiscal financing

Note: The numbers are fractions of the present value of increases in government consumption. Each column sums to 1.

contribution of the real interest rate. After a negative government consumption shock occurs, consumption drops and grows back to the steady state. Therefore, the real interest rate is higher along this path.

In our model economy where inflation erodes the collateral value, inflation only finances 55.65% of higher government consumption, half of the size in the frictionless model. Meanwhile, increases in tax revenue account for 51.87%. The negative contribution of the real interest rate is smaller than in the frictionless model, because the lower value of real debt dampens the rise in the real interest rate due to the dearth of collateral.

Volatility of inflation and tax rate. In standard models, optimal policy displays large inflation volatility, because state-contingent inflation is purely a fiscal buffer (Chari et al., 1991). Indeed, the first column of Table 3 shows that the quarterly volatility of the labor tax rate is near zero, while that of the inflation rate is 0.96%. In contrast, in the model with financial frictions, the standard deviation of inflation is dampened by half, and the labor tax rate becomes much more volatile. In this sense, our model provides a new explanation in addition to price stickiness for the undesirability of volatile inflation.

	Frictionless model	Friction model
Inflation	0.96%	0.47%
Tax rate	0.02%	0.35%

Table 3: Standard deviation of tax rate and inflation (quarterly)

3.3 Comparison with alternative policies

The optimal policy is determined by balancing the tradeoff between tax distortions and the cost of inflation. To illustrate the tradeoff, we compare the Ramsey optimal policy with two sets of alternative policies. One is a constant-tax policy, where the labor tax rate is set constant at the steady-state value and the government adjusts state-contingent inflation to satisfy its inter-temporal budget constraint. The other is a zero-inflation economy where state-contingent inflation is shut down. The three economies share the same steady state.

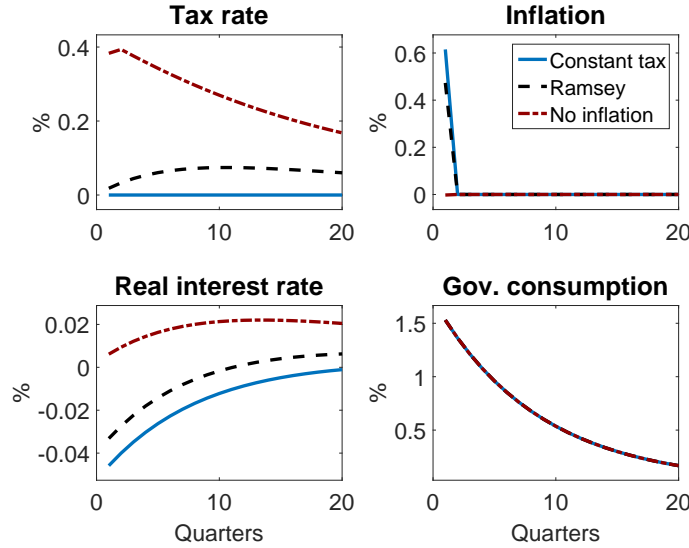


Figure 3: Optimal policy responses to one standard-deviation fiscal shock.

Note: The shock is 1.53% with the first-order autocorrelation coefficient of 0.89. The black dashed line represents the Ramsey optimal policy. The blue line represents a policy where the labor tax rate is set constant at the steady-state value. The red dashed-dotted line represents a policy where state-contingent inflation is ruled out.

Figure 3 shows the impulse responses in these three economies. In the constant-tax economy, inflation rate increases by 0.62% in the initial period, compared to 0.47% in the optimal policy. Higher inflation causes a more severe dearth of collateral in the economy, which increases the liquidity premium and reduces the real interest rate. A lower interest rate in turn alleviates the fiscal stress, preventing the inflation rate from increasing further. In the zero-inflation economy, the increase in the labor tax rate amounts to 0.38% in the initial

quarter and remains higher than 0.20% after 16 quarters. As the amount of government debt increases to smooth tax distortions, the real interest rate rises and exacerbates the fiscal stress.

3.4 Sensitivity analysis

Figure 4 shows the sensitivity of results as the key parameter ξ varies.¹⁸ As in the last panel, a higher value of ξ relaxes the collateral constraint, and the optimal size of the government debt in the steady state declines. To compare economies with and without collateral constraints, for each value of ξ , we set the steady-state debt-to-GDP ratio in the frictionless model to be the same as in the model with frictions.

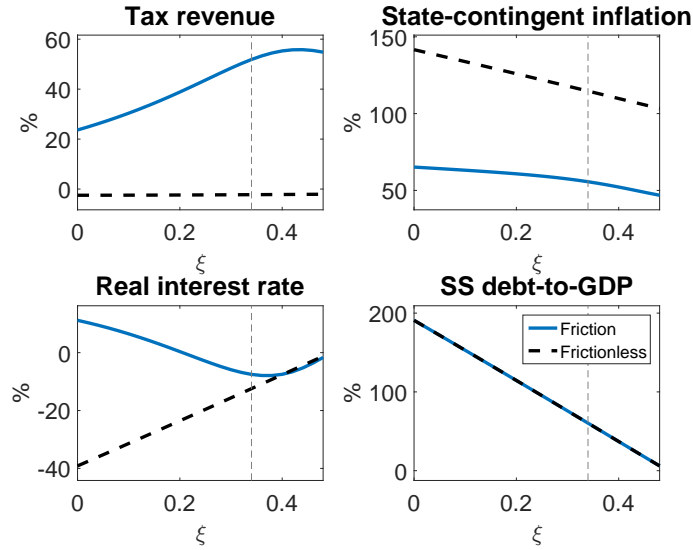


Figure 4: Sensitivity analysis for ξ .

Note: The value of ξ is irrelevant for the frictionless economy, so for each value of ξ , we set the debt-to-GDP ratio in the frictionless economy to be the same as in the economy with collateral constraints. The vertical dashed line shows the benchmark value $\xi = 0.34$.

In the frictionless model, the contribution of tax revenues is always near zero, and the contribution of the real interest rate increases as debt-to-GDP ratio decreases (ξ increases). A negative fiscal shock raises the real interest rate and debt is more costly to serve, and

¹⁸Appendix A.7 shows additional sensitivity analysis.

this effect is weaker when the debt-to-GDP ratio is smaller. As the contribution of the real interest rate increases, that of inflation declines.

In the model with frictions, the contribution of inflation also declines as ξ increases, ranging from 47% to 65%. Most importantly, for all values of ξ , inflation makes a far smaller contribution in the economy with collateral constraints than that without. The contribution of the real interest rate is nonlinear because of two forces at play. On the one hand, the effect of the liquidity premium gets weaker as ξ increases and the economy is less constrained, which lowers the contribution of the real interest rate. On the other hand, the steady-state debt-to-GDP gets smaller as ξ increases, which attenuates the negative contribution of the real interest rate.

4 Introducing price stickiness and long-term government debt

Government debt has one-period maturity in the previous section. Only inflation in the current period can erode the real return on government debt in response to a fiscal shock. In this section, we introduce price stickiness and long-term government debt into the model. As another cost of inflation, price stickiness makes abrupt changes in price level particularly costly. Long-term government debt allows both current and future inflation to affect the real return on debt. Therefore, the Ramsey optimal policy features modest but persistent inflation, consistent with the data.

4.1 Model

Price stickiness and retail firms. In order to incorporate price stickiness into the model, we introduce a continuum of retail firms. Retail firms are monopolistic competitors. They buy goods from competitive firms owned by bankers, differentiate these goods costlessly, and sell them to households. The monopoly power of retail firms allows them to set prices above

marginal costs. Profits from retail activity are rebated lump-sum to households.¹⁹

The final goods used in household consumption and investment are aggregated from the differentiated goods using constant elasticity of substitution (CES) technology. The household chooses their demand of each type of good j :

$$y_{j,t} = y_t \left(\frac{P_{j,t}}{P_t} \right)^{-\nu}, \quad (20)$$

where ν measures the elasticity of substitution across goods sold by retail firms. P_t denotes the aggregate nominal price level, and $P_{j,t}$ denotes the nominal price of type- j good.

We introduce price stickiness through a Rotemberg-style price adjustment costs; to adjust nominal price $P_{j,t}$, retail firm j pays $\frac{\psi}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2$ units of final goods. Retail firm j sets price $\{P_{j,s}\}_{s \geq t}$ to maximize the expected discounted sum of real profits that it rebates to the household:

$$\max_{P_{j,s}} \mathbb{E}_t \sum_{s \geq t} \Lambda_{t,s} v_s^R \equiv \mathbb{E}_t \sum_{s \geq t} \Lambda_{t,s} \left[\frac{P_{j,s}}{P_s} y_{j,s} - m_s y_{j,s} - \frac{\psi}{2} \left(\frac{P_{j,s}}{P_{j,s-1}} - 1 \right)^2 \right],$$

subject to the demand function for good j in equation (20). $\Lambda_{t,s}$ is the household's real stochastic discount factor, and m_t is the real price (in the units of final goods) to purchase goods from bankers' firms. In other words, it is the real marginal cost for a retailer to produce differentiated goods j .

We focus on a symmetric equilibrium where each retail firm j sets the same price $P_{j,t}$ and $P_{j,t} = P_t$ for all j . The optimality condition of retail firms takes the form of the New

¹⁹The separation of competitive and flexible-price firms held by bankers from sticky-price retail firms follows the approach of [Bernanke et al. \(1999\)](#). Directly introducing price stickiness to the firms held by bankers will destroy the tractability of the model, because these firms receive idiosyncratic productivity shocks. High-productivity firms would set a lower price and vice versa. Therefore, we would need to keep track of the history and the cross-sectional distribution of prices.

Keynesian (NK) Phillips curve.²⁰

$$[\nu m_t - (\nu - 1)] y_t - \psi [(\pi_t - 1)\pi_t - \mathbb{E}_t \Lambda_{t,t+1} (\pi_{t+1} - 1) \pi_{t+1}] = 0. \quad (21)$$

In the flexible-price setting where $\psi = 0$, equation (21) reduces to $1 = \frac{\nu}{\nu-1} m_t$.

The social resource constraint now takes into account the real adjustment cost from changing prices:

$$c_t + g_t + a_t + \frac{\psi}{2} \left(\frac{P_t}{P_{t-1}} - 1 \right)^2 = y_t + (1 - \delta) a_{t-1}.$$

Long-term nominal government debt. Following the literature (e.g., [Arellano and Ramanarayanan, 2012](#); [Hatchondo et al., 2016](#)), we model long-term nominal government debt as a security paying an infinite stream of nominal coupons, which decreases at a constant rate η . In particular, a bond issued in period t promises to pay $(1 - \eta)^{s-1}$ dollars in period $t + s$, with $s \geq 1$. The exogenous parameter η dictates the average maturity of bonds. This method of modeling long-term debt allows us to study long-maturity bonds without increasing the dimensionality of the state space.

As in previous sections, we denote the units of nominal government debt by B_t and the nominal price of debt by Q_t^B . The household budget constraint in real terms is given by:

$$c_t + a_t + \frac{Q_t^B B_t}{P_t} = [\sigma v_t^H + (1 - \sigma) v_t^L] + v_t^R + (1 - \tau_t) w_t h_t + \frac{1 + (1 - \eta) Q_t^B}{P_t} B_{t-1} + q_t a_{t-1}. \quad (22)$$

This constraint differs from the one in the baseline model (equation 5) in terms of the retailers' profits v_t^R and the long maturity of government debt η . The collateral constraint

²⁰Log-linearizing equation (21), we get the more familiar-looking NK Phillips curve:

$$\tilde{\pi}_t = \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \frac{\nu - 1}{\psi} \bar{y} \tilde{m}_t,$$

where $\tilde{\pi}_t$ is the log-linearized inflation rate, and \bar{y} is the steady-state value of output.

(4) becomes:

$$k_t^s \leq \frac{1}{q_t - \xi} \left[q_t a_{t-1} + \frac{1 + (1 - \eta)Q_t^B}{P_t} B_{t-1} \right]. \quad (23)$$

The firm's profit function is $v_t^s = m_t z^s F(k_t^s, n_t^s) - w_t n_t^s - [q_t - (1 - \delta)] k_t^s$, $s \in \{H, L\}$. The household maximizes its utility subject to budget constraint (22) and the collateral constraint (23).

Government policies need to satisfy the budget constraint:

$$\tau_t w_t h_t + \frac{Q_t^B B_t}{P_t} = \frac{1 + (1 - \eta)Q_t^B}{P_t} B_{t-1} + g_t.$$

Long-term debt allows the government to adjust the real return on debt not only through current inflation but also through inflation in future periods. The amount of real government debt is $\frac{Q_t^B B_t}{P_t}$, and the real holding-period return on government debt is $r_t^b \equiv \frac{1 + (1 - \eta)Q_t^B}{Q_{t-1}^B \pi_t}$. If $\eta = 1$ and government debt is one-period debt, the only way to adjust the real return ex post is through inflation in the current period π_t , but abrupt changes in π_t incur the adjustment costs. In contrast, if government debt has a long maturity, that is, $\eta < 1$, adjustment in the real return ex post can be engineered through changes in bond price Q_t^B , which depends on inflation in future periods. In other words, long-term debt helps the Ramsey government to achieve the desired adjustment in the ex post real return at less cost.

4.2 Numerical analysis

We solve for the Ramsey optimal policy in the model. Parameters that have appeared in the baseline model take the same values in Table 1. The two parameters new to the sticky-price setting are the elasticity of substitution ν and the degree of price stickiness ψ . We set these parameters to values estimated from U.S. data in Christiano et al. (2005). In particular, we set $\nu = 6$, so the markup of retail firms is 20%. We set ψ to replicate in a linearized setup the slope of the Phillips curve derived using Calvo stickiness with an average duration of

prices of three quarters.²¹ We recalibrate ξ such that steady-state debt-to-GDP ratio is 61%.

The steady state of the Ramsey policy features zero price inflation; that is, $\bar{\pi} = 1$. The reason is that there is no gain from inflation in the non-stochastic steady state in the absence of any fiscal shocks. On the other hand, any deviation from zero inflation leads to a positive adjustment cost in real resources.

Fiscal financing decomposition. We perform a similar decomposition exercise as in the section 3, using the linear approximation of the inter-temporal government budget constraint. The decomposition follows:

$$\begin{aligned}
\underbrace{\sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{g}}{\bar{b}} \tilde{g}_s}_{\text{government consumption}} &= \underbrace{\sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{T}}{\bar{b}} \tilde{T}_s}_{\text{tax revenue}} + \underbrace{\frac{1}{\bar{\pi}} \hat{\pi}_t}_{\text{current inflation}} + \underbrace{\sum_{s=t+1}^{\infty} \frac{(1-\eta)^{s-t}}{(\bar{r}^b)^{s-t} \bar{\pi}} \hat{\pi}_s}_{\text{future inflation}} \\
&\quad - \underbrace{\sum_{s=t+1}^{\infty} \frac{1 - (1-\eta)^{s-t}}{(\bar{r}^b)^{s-t+1}} \hat{r}_s^b}_{\text{real interest rate}}.
\end{aligned} \tag{24}$$

See Appendix A.6.2 for the derivations. The present value of higher government consumption is financed by changes in current inflation, future inflation, taxes, and the real interest rate. Longer maturity of debt (smaller η) affects the decomposition in two ways. First, the contribution of inflation in future periods becomes larger, because it reduces the real value of coupon payments in future periods. Second, the contribution of the real interest rate becomes smaller, as the government only needs to roll over a smaller fraction of debt in each period.

The results of the financing decomposition are reported in Table 4. The first two columns compare the flexible-price economy ($\psi = 0$) and the economy with price stickiness and one-period government debt ($\psi > 0$ and $\eta = 1$). In the second economy, the government can only increase the inflation rate in the current period to reduce the real return on debt,

²¹The slope of the Phillips curve in a quarterly Calvo price-setting model is $\frac{(1-\kappa)(1-\beta\kappa)}{\kappa}$, where κ is the probability of not being able to re-optimize price (Galí, 2009). $\kappa = 0.667$ is consistent with the average duration of the wage contract being three quarters. The slope in the Rotemberg model in this paper is $\frac{(\nu-1)\bar{y}}{\psi}$, where \bar{y} is the steady-state value of output. We set $\frac{(\nu-1)\bar{y}}{\psi} = \frac{(1-\kappa)(1-\beta\kappa)}{\kappa}$.

but it is costly to do so due to the presence of the price adjustment cost. Therefore, the contribution of current inflation reduces to 13.60%, in contrast with 64.53% in the flexible-price economy.²² In addition, the negative contribution of the real interest rate becomes more severe (-43.66%) in the sticky-price model. As the government cannot monetize debt adequately, there exists too much collateral in the economy, and the liquidity premium is low, causing a higher interest rate. The small contributions of inflation and the negative contribution of the real interest rate imply a large contribution of taxes.

	Flexible price 3-month debt	Sticky price 3-month debt	Sticky price 5-year debt	Sticky price 10-year debt
Tax revenue	45.68%	130.06%	97.93%	85.25%
Current inflation	64.53%	13.60%	5.43%	3.00%
Future inflation	0.00%	0.00%	25.95%	33.53%
Real interest rate	-10.21%	-43.66%	-29.31%	-21.78%

Table 4: Decomposition of fiscal financing

Note: All four economies are associated with collateral constraints. They only differ in the degree of price stickiness and the maturity of government debt.

Inflation plays a much larger role when government debt has a long maturity. Column 3 of Table 4 shows the fiscal decomposition when the average duration of government debt is five years, which is consistent with the U.S. data.²³ The contributions of current and future inflation add up to 31.38%, in contrast with 13.60% in the short-term debt economy. Future inflation plays a much more important role relative to inflation in the current period (25.95% vs. 5.43%). In addition, the negative contribution of the real interest rate becomes smaller, as the government only rolls over a small fraction of debt. Column 4 presents the result when government debt has a longer duration of 10 years, consistent with the U.K. data. The total contribution of inflation is even larger in that case (36.53%).

²²The results in column 1 differ from those in the previous section due to the presence of monopolistic competition; that is, $\nu < \infty$.

²³We use the concept of Macaulay duration, which in the steady state is given by:

$$D = \frac{1 + \bar{r}^b}{\eta + \bar{r}^b}.$$

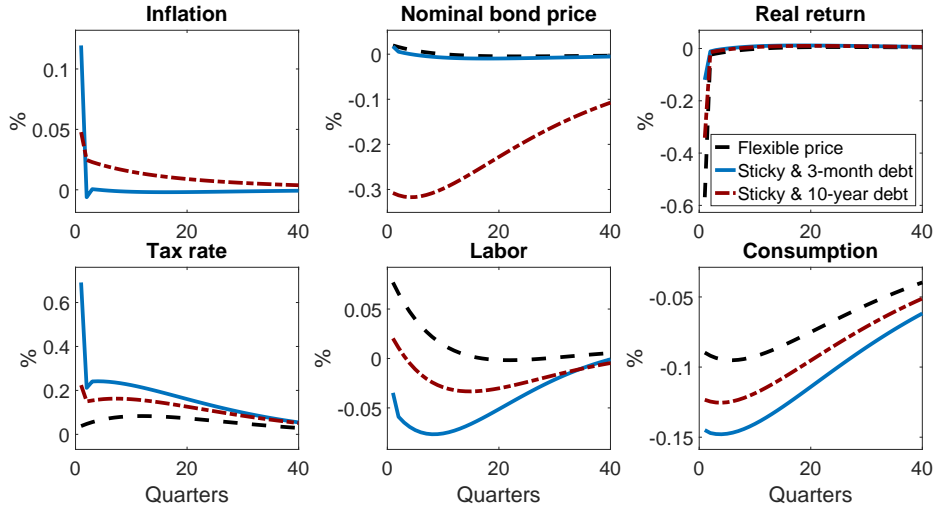


Figure 5: Optimal policy responses to one standard-deviation fiscal shock.

Note: The shock is 1.53% with the first-order autocorrelation coefficient of 0.89.

Figure 5 further illustrates the model dynamics by showing the optimal policy responses to a one-standard-deviation fiscal shock. When the government issues short-term debt, the inflation rate rises by 0.12% in the same quarter as the fiscal shock occurs, compared to 0.57% in the flexible-price economy.²⁴ When the government can issue long-term debt, the optimal response of inflation is even smaller initially (0.03%) but much more persistent in later periods. The cumulative inflation in the 10 years after the occurrence of the fiscal shock is 0.38%. Inflation leads to a large decline in the nominal bond price and facilitates the reduction of real debt return in the period when the shock occurs. As a result, the increase in the tax rate is greatly dampened. In terms of real allocations, labor and consumption in the economy with long-term debt become much more similar to that in the flexible-price economy than to that in the economy with short-term debt.

²⁴Inflation in the flexible-price economy is not plotted for the consideration of presentation, since it has a much larger magnitude than in the other two economies.

4.3 Application: war financing

As an application, we use the model to study the financing of the wars in Afghanistan and Iraq by the U.S. government. The total appropriations for these wars in 2001–2013 amount to \$1.54 trillion (Crawford, 2014). As shown in the left panel of Figure 6, the budgetary costs of the wars amount to 7% of total government consumption at the peak.

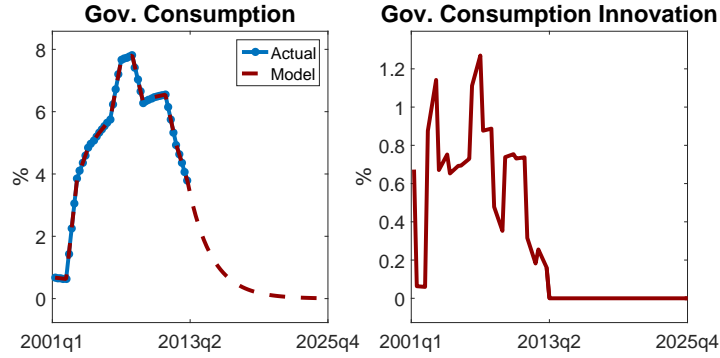


Figure 6: Increases in government consumption caused by wars in Afghanistan and Iraq.

We use the model to study optimal fiscal and monetary policy response to increases in war appropriations. We simulate the model with government consumption shocks that match war costs in the data. To do this, we assume that the government consumption process follows the AR(1) process we estimated in the previous sections. We then calculate the series of shocks that makes the government consumption process match the data in 2001–2013. We assume that after 2013, there are no more shocks, and government consumption declines at the rate in the AR(1) process. The right panel in Figure 6 shows the series of shocks to the government consumption process.

We calibrate the model to the features of the U.S. economy before the wars. The debt-to-GDP ratio in 2000 is 54.6%. The average maturity of government debt is 5.8 years, according to the Department of the Treasury. We assume that the economy is in the steady state before the series of war shocks arrive.

Figure 7 displays the prescriptions of three models for war financing. The black dashed

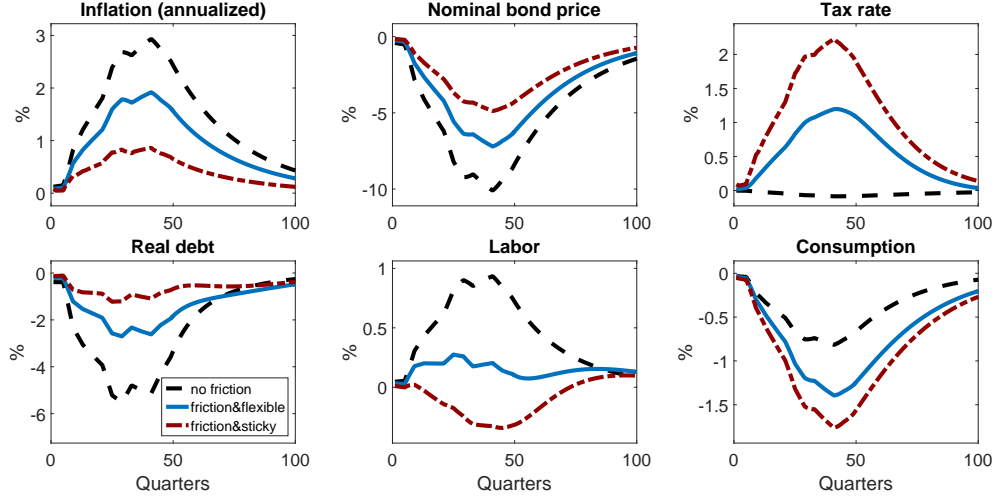


Figure 7: War financing.

line represents the model without financial friction and price stickiness.²⁵ The government solely relies on inflation to adjust the real debt value, and the tax rate remains quite constant. The average annual inflation rate in 25 years is 1.52%. The blue line shows the economy with the existence of the collateral constraint. Financial friction dampens the increase in the inflation rate; the average increase in the annual inflation rate is now 0.99%. At the same time, the government raises the labor tax rate by an average of 0.57 percentage point from 35.72 percentage points in the steady state. Finally, the red line represents the economy with both financial friction and prices stickiness, which is the most realistic and preferred calibration. Price stickiness further reduces the use of inflation to 0.44% on average, and the rise in the labor tax rate is now 1.08 percentage points.

5 Conclusion

We introduce into the Ramsey optimal fiscal and monetary policy framework the bank balance sheet costs of inflation that reduce the banks' net worth and tighten their financing

²⁵Strictly speaking, in this model where prices are perfectly flexible and government debt has a long maturity, the optimal path of inflation is indeterminate. This is because the government is indifferent between using current and future inflation to adjust the real value of debt. In the simulation, we determine the optimal path of inflation by setting a tiny degree of price stickiness (10^{-4}).

constraints. We study how a Ramsey planner trades off the distortionary effect of a linear labor tax and the bank balance sheet costs of inflation.

Our main finding is that, quantitatively, inflation plays a much smaller role in fiscal financing compared with standard models, and it is also much less volatile. Thus, the bank balance sheet costs of inflation substantially change the optimal fiscal and monetary policy prescriptions.

We also introduce price stickiness and a long maturity of government debt into the model. The optimal response of inflation becomes modest and persistent. The maturity of government debt has a great impact on the policy prescriptions. The role of inflation in optimal fiscal financing increases with the maturity of government debt.

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A Appendix

A.1 Proof of Lemma 1

Part (a). Let $x_t = \frac{k_t^H}{a_{t-1}}$ be the amount of capital used by the high-productivity banks as a fraction of the aggregate capital stock. Then we have:

$$\frac{k_t^L}{a_{t-1}} = \frac{1 - \sigma x_t}{1 - \sigma}.$$

There is no friction in the labor market, so the marginal product of labor is equalized between high- and low-productivity banks, i.e.,

$$\theta z^H (k_t^H)^\alpha (n_t^H)^{\theta-1} = \theta z^L (k_t^L)^\alpha (n_t^L)^{\theta-1}.$$

One can solve for the ratio of employment at two types of banks n_t^H/n_t^L from the above equation. Together with the labor market clear condition $\sigma n_t^H + (1 - \sigma) n_t^L = h_t$, we obtain the fraction of labor used by each type of banks:

$$\frac{n_t^H}{h_t} = \frac{(z^H)^{\frac{1}{1-\theta}} (x_t)^{\frac{\alpha}{1-\theta}}}{\Gamma(x_t)^{\frac{1}{1-\theta}}}, \quad (25)$$

and

$$\frac{n_t^L}{h_t} = \frac{(z^L)^{\frac{1}{1-\theta}} \left(\frac{1-\sigma x_t}{1-\sigma} \right)^{\frac{\alpha}{1-\theta}}}{\Gamma(x_t)^{\frac{1}{1-\theta}}}. \quad (26)$$

Given the allocations of capital $(k_t^H/a_{t-1}, k_t^L/a_{t-1})$ and the allocations of labor $(n_t^H/h_t, n_t^L/h_t)$, the aggregate production function can be written as:

$$\begin{aligned} y_t &= \sigma z^H (k_t^H)^\alpha (n_t^H)^\theta + (1 - \sigma) z^L (k_t^L)^\alpha (n_t^L)^\theta \\ &= \left[\sigma z^H \left(\frac{k_t^H}{a_t} \right)^\alpha \left(\frac{n_t^H}{h_t} \right)^\theta + (1 - \sigma) z^L \left(\frac{k_t^L}{a_t} \right)^\alpha \left(\frac{n_t^L}{h_t} \right)^\theta \right] a_t^\alpha h_t^\theta \equiv \Gamma(x_t) a_{t-1}^\alpha h_t^\theta. \end{aligned}$$

Part (b)-(d). In first order conditions (8) and (9), substitute for k_t^H , k_t^L , n_t^H , and n_t^L

using $k_t^H = x_t a_{t-1}$, $k_t^L = \frac{1-\sigma x_t}{1-\sigma} a_{t-1}$, and equations (25) and (26). Then we can derive w_t , q_t , and μ_t^H as a function of a_{t-1} , h_t , and x_t .

A.2 Proof of Lemma 2

Proof of the “only if”. We need to show that the set of competitive equilibrium conditions implies the set of constraints in Lemma 2. It is obvious that the social resource constraint (2), the Euler equation (14), $\mu^H(\cdot) \geq 0$, and the complementary slackness condition are satisfied by the competitive equilibrium conditions. It remains to show that the implementability constraint (17) and the collateral constraint (18) hold.

Implementability condition. Using equation (7) to substitute for $(1 - \tau_t)w_t h_t$ in the household budget constraint (5), we have:

$$c_t + a_t + \frac{B_t}{P_t} = [\sigma v_t^H + (1 - \sigma)v_t^L] - \frac{U_{h,t}}{U_{c,t}} h_t + q_t a_{t-1} + \frac{R_{t-1}^B B_{t-1}}{P_t}.$$

From the definition of the profit function (6) we obtain:

$$\sigma v_t^H + (1 - \sigma)v_t^L = y_t - w_t h_t - [q_t - (1 - \delta)]a_{t-1}.$$

The labor and capital demand conditions (8)-(9) imply that

$$w_t h_t = \theta y_t,$$

and

$$[q_t - (1 - \delta)] a_{t-1} = \alpha y_t - \sigma \mu_t^H k_t^H = \alpha y_t - \frac{\sigma \mu_t^H}{q_t - \xi} \left(\frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1} \right),$$

where the last equality holds no matter the collateral constraint for the high-productivity

bankers binds or not.²⁶ Due to frictions in capital allocations, the share of capital income measured at market price of capital q_t is smaller than α when the collateral constraint strictly binds for the high type (i.e., $\mu_t^H > 0$).

The total profits of firms become:

$$\sigma v_t^H + (1 - \sigma)v_t^L = (1 - \alpha - \theta)y_t + \frac{\sigma \mu_t^H}{q_t - \xi} \left(\frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1} \right).$$

Using the expression above to substitute for total profits $\sigma v_t^H + (1 - \sigma)v_t^L$ in the household budget constraint, we have:

$$\begin{aligned} & [U_{c,t}c_t + U_{h,t}h_t - U_{c,t}(1 - \alpha - \theta)y_t] + U_{c,t} \left(\frac{B_t}{P_t} + a_t \right) \\ &= U_{c,t} \left(1 + \frac{\sigma \mu_t^H}{q_t - \xi} \right) \left(\frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1} \right). \end{aligned} \quad (27)$$

Taking conditional expectation in date $t - 1$ on both sides of the equation and using the Euler equations (14) and (15), we arrive at the flow implementability condition in equation (17).

Collateral constraint. Combining the government budget constraint (3) and equation (16), we can express the outstanding value of debt at the beginning of period t by:

$$r_t^b b_{t-1} = \theta y_t + \frac{U_{h,t}}{U_{c,t}} h_t + b_t - g_t.$$

Substituting for $r_t^b b_{t-1}$ in the collateral constraint by using the equation above, we arrive at the form of collateral constraint in equation (18).

²⁶To see this, if the collateral constraint binds for the high-productivity bankers, we have:

$$k_t^H = \frac{1}{q_t - \xi} \left(\frac{R_{t-1}^B B_{t-1}}{P_t} + q_t a_{t-1} \right).$$

If the collateral constraint does not bind, we have:

$$\mu_t^H = 0.$$

Proof of the “if”. We need to show that if allocations $\{a_t, h_t, x_t, c_t, b_t\}_{t \geq 0}$ satisfy the set of constraints in Lemma 2, we can construct prices $\{w_t, q_t, \mu_t^H\}_{t \geq 0}$ and policies $\{\tau_t, r_t^b\}_{t \geq 0}$ that satisfy all the competitive equilibrium conditions together with the set of allocations.

The wage rate w_t , the price of capital q_t , the multiplier on high type’s collateral constraint μ_t^H , and the tax rate τ_t are implied by equations (11), (12), (13) and (16), respectively. r_t^b can be backed out through the government budget constraint (3):

$$r_t^b = \frac{(1 - \tau_t) w_t h_t + b_t - g_t}{b_{t-1}}.$$

It is straightforward to check that allocations $\{a_t, h_t, x_t, c_t, b_t\}_{t \geq 0}$, prices $\{w_t, q_t, \mu_t^H\}_{t \geq 0}$ and policies $\{\tau_t, r_t^b\}_{t \geq 0}$ satisfy the competitive equilibrium conditions.

A.3 Proof of Lemma 3

In the economy without the collateral constraint, a household’s decision problem is to choose $\{k_t^s, n_t^s, h_t, c_t, a_t, B_t\}_{t=0}^\infty$ to maximize utility (1), subject to the end-of period budget constraint (5).

The set of first order conditions is similar to that in the problem with the collateral constraint, and the aggregate economy can be characterized in the same manner. The difference is that capital allocations between the two types of banks are always optimal; in other words, $x_t = x^*$.

Given the initial household asset positions a_{-1} and $R_{-1}^B B_{-1}$, and the process of government consumption shocks $\{g_t\}_{t \geq 0}$, the *competitive equilibrium* can be summarized by a set of allocation $\{y_t, a_t, h_t, c_t, B_t\}_{t \geq 0}$, prices $\{q_t, w_t, P_t\}_{t \geq 0}$, and fiscal and monetary policies $\{\tau_t, R_t^B\}_{t \geq 0}$ satisfying (3), (5), (7), (10)-(12), and (14)-(15); x_t is set to x^* and μ_t^H set to 0 when they show up in these equations.

To prove Lemma 3, we can follow the steps in appendix A.2 to show the equivalence of the competitive equilibrium conditions to the conditions in the primal approach.

A.4 Proof of Proposition 1

The Ramsey problem without financial frictions is formulated in Lemma 3. When the utility function is quasi-linear, it can be written as:

$$\max_{a_t, h_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Gamma(x^*) F(a_{t-1}, h_t) - a_t + (1 - \delta)a_{t-1} - g_t - \chi \frac{h_t^{1+\epsilon}}{1+\epsilon} \right],$$

subject to:

$$\begin{aligned} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [(\alpha + \theta) \Gamma(x^*) F(a_{t-1}, h_t) + (1 - \delta)a_{t-1} - a_t - g_t - \chi h_t^{1+\epsilon}] &= r_0^b b_{-1} + q_0 a_{-1}, \quad [\omega] \\ \beta \mathbb{E}_t [1 - \delta + \Gamma(x^*) F_a(a_t, h_{t+1})] &= 1. \quad [\gamma_t] \end{aligned} \tag{28}$$

In the formulation above, we have substituted for consumption c_t using the resource constraint and substituted for capital price q_t using the capital demand condition (12). The implementability constraint has been written in the time-0 form by iterating the flow implementability constraint (17).

The first-order condition for capital a_t ($t \geq 0$) is:

$$\begin{aligned} (1 + \omega) [-1 + \beta(1 - \delta)] + \beta \mathbb{E}_t [1 + (\alpha + \theta) \omega] \Gamma(x^*) F_a(a_t, h_{t+1}) \\ + \beta \gamma_t \mathbb{E}_t \Gamma(x^*) F_{aa}(a_t, h_{t+1}) = 0. \end{aligned} \tag{29}$$

The first-order condition for h_t ($t \geq 1$) is:

$$[1 + (\alpha + \theta) \omega] \Gamma(x^*) F_h(a_{t-1}, h_t) - [1 + (1 + \epsilon) \omega] h_t^\epsilon + \gamma_{t-1} \Gamma(x^*) F_{ah}(a_{t-1}, h_t) = 0. \tag{30}$$

Government consumption shock g_t enters neither the first order conditions of the Ramsey planner (29)–(30) nor the Euler equation of the household (28), which suggests that h_t , a_t are independent of g_t . That is, $h_t = h^*$, $a_t = a^*$, and $\gamma_t = \gamma^*$, where h^* , a^* , and γ^* are

constant and independent of time t and state g^t . Then the first-order conditions become:

$$(1 + \omega) [-1 + \beta(1 - \delta)] + \beta [1 + (\alpha + \theta)\omega] \Gamma(x^*) F_a(a^*, h^*) + \beta \gamma^* \Gamma(x^*) F_{aa}(a^*, h^*) = 0,$$

and

$$[1 + (\alpha + \theta)\omega] \Gamma(x^*) F_h(a^*, h^*) - [1 + (1 + \epsilon)\omega] h^{*\epsilon} + \gamma^* \Gamma(x^*) F_{ah}(a^*, h^*) = 0.$$

The Euler equation of capital becomes:

$$\beta [1 - \delta + \Gamma(x^*) F_a(a^*, h^*)] = 1.$$

Therefore, a^* , h^* , and γ^* are pinned down by the above three equations as functions of parameters and the multiplier ω , and ω is determined by ensuring that the implementability constraint hold. It immediately follows that the tax rate τ_t is also constant across states:

$$\tau_t = 1 - \frac{\chi h^{*\epsilon}}{\Gamma(x^*) F_h(a^*, h^*)}.$$

A.5 Accuracy of solution

The quantitative results presented in section 3 are based on a log-linear approximation to the first-order conditions of the Ramsey problem. For the simplified model under the assumption of quasi-linear utility, we have computed numerical solutions using global methods. Therefore, we can evaluate the accuracy of the log-linear solution for the case with quasi-linear utility.

Table 5 shows the maximum percentage deviation of the linear solution from the global solution. It shows that the quantitative results obtained using the global solution and the log-linear approximation are very similar. The most noticeable differences concern the capital and the labor tax rate, with a percentage difference of around 0.72%.

Real interest rate	Tax rate	Debt issuance	Labor
0.08%	0.73%	0.08%	0.18%
Capital	x	Output	
0.72%	0.02%	0.14%	

Table 5: Comparing the linear solution and the global solution.

The table reports percentage deviation of the linear solution from the global solution.

Next, we compare the first- and the second-order approximations to the Ramsey problem with general utility functions. Figure 8 reports the responses of government policies and allocations to a one standard-deviation government consumption shock in the first- and second-order approximations. At least for the size of government consumption shocks experienced by the U.S. economy, the first- and second-order approximations are very similar.

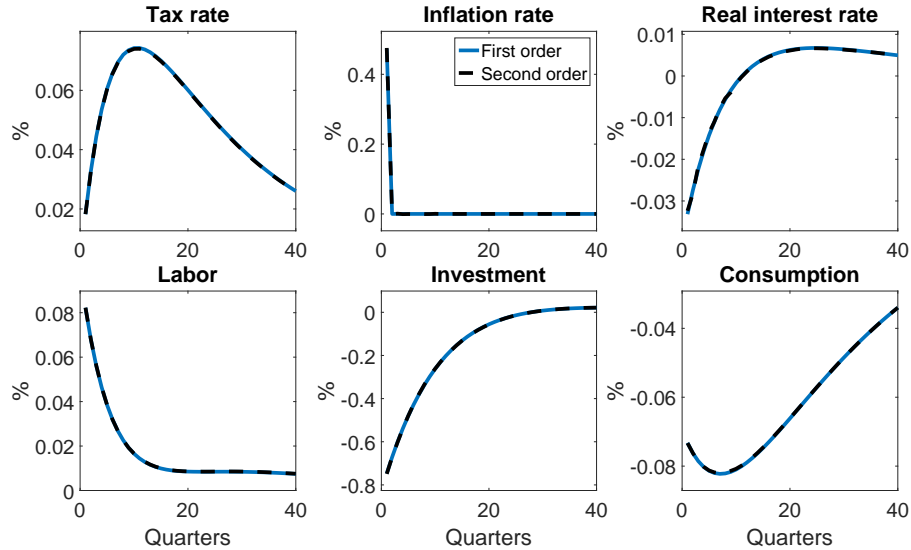


Figure 8: Comparing first- and second-order approximations.

Note: The figure comparing the responses of policies and allocations in the first- and second-order approximations. The shock is a one standard-deviation government consumption shock.

A.6 Derivations of the fiscal financing decompositions

A.6.1 One-period nominal government debt

In this subsection, we derive equation (19). We consider the situation where the government consumption shock occurs in date t when the economy is at the steady state. We start from the government budget constraint:

$$T_t + b_t = r_t^b b_{t-1} + g_t,$$

where $T_t = \tau_t w_t h_t$ is the labor tax revenue. By linearizing this equation we get:

$$\tilde{b}_{t-1} = \frac{1}{\bar{r}^b} \tilde{b}_t - \frac{1}{\bar{r}^b} \hat{r}_t^b - \frac{\bar{g}}{\bar{r}^b \bar{b}} \tilde{g}_t + \frac{\bar{T}}{\bar{r}^b \bar{b}} \tilde{T}_t,$$

where \bar{X} , \hat{X} , and \tilde{X} denote the steady state level, the level deviation from the steady state, and the percentage deviation from the steady state of variable X , respectively. By iterating this equation forward, we get:

$$\tilde{b}_{t-1} = \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \left(-\hat{r}_s^b - \frac{\bar{g}}{\bar{b}} \tilde{g}_s + \frac{\bar{T}}{\bar{b}} \tilde{T}_s \right).$$

As the shock arrives in period t , $\tilde{b}_{t-1} = 0$. Therefore:

$$\sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{g}}{\bar{b}} \tilde{g}_s = \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{T}}{\bar{b}} \tilde{T}_s - \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \hat{r}_s^b. \quad (31)$$

In period t when the shock occurs, the real return of debt r_t^b follows the Fisher equation:

$$r_t^b = \frac{R_{t-1}^B}{\pi_t}.$$

By linearizing the Fisher equation and using the fact that the nominal interest rate is pre-determined ($\hat{R}_{t-1}^B = 0$), we get:

$$\hat{r}_t^b = -\frac{\bar{r}^b}{\bar{\pi}} \hat{\pi}_t.$$

Combining this equation with equation (31), we arrive at the fiscal financing decomposition condition (19):

$$\begin{aligned} \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{g}}{\bar{b}} \tilde{g}_s &= \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{T}}{\bar{b}} \tilde{T}_s - \frac{\hat{r}_t^b}{\bar{r}^b} - \sum_{s=t+1}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \hat{r}_s^b \\ &= \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{T}}{\bar{b}} \tilde{T}_s + \frac{1}{\bar{\pi}} \hat{\pi}_t - \sum_{s=t+1}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \hat{r}_s^b. \end{aligned}$$

A.6.2 Long-term nominal government debt

In this subsection, we derive equation (24). When the government debt has long maturity, equation (31) still holds, but the period- t return on nominal debt becomes:

$$r_t^b = \frac{1 + (1 - \eta)Q_t^B}{Q_{t-1}^B \pi_t}. \quad (32)$$

Linearizing this equation and using the fact that $\tilde{Q}_{t-1}^B = 0$, we have:

$$\hat{r}_t^b = \frac{1 - \eta}{\bar{\pi}} \tilde{Q}_t^B - \frac{\bar{r}^b}{\bar{\pi}} \hat{\pi}_t,$$

which shows that the real return on debt depends on the nominal bond price and the inflation rate in the current period. The nominal bond price Q_t^B is in turn a function of the future real interest rates and inflation rates, which can be shown by iterating \tilde{Q}_t^B forward using a linearized version of equation (32):

$$\tilde{Q}_t^B = \sum_{s=t+1}^{\infty} \left(\frac{1 - \eta}{\bar{r}^b \bar{\pi}} \right)^{s-t-1} \left(-\frac{\hat{\pi}_s}{\bar{\pi}} - \frac{\hat{r}_s^b}{\bar{r}^b} \right).$$

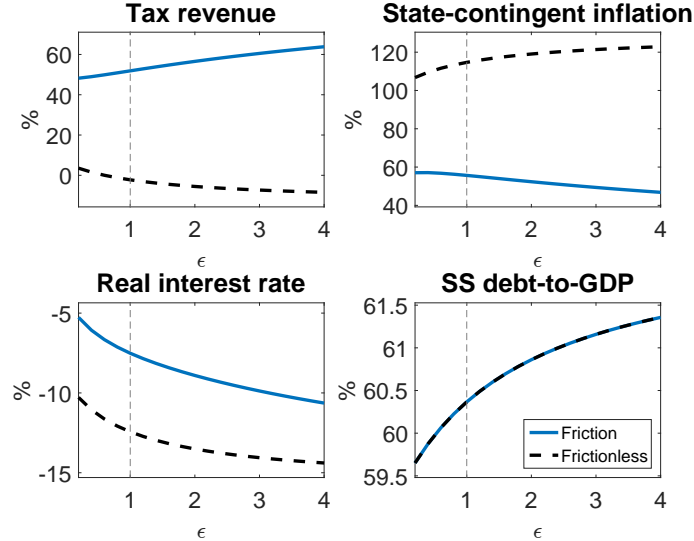
Therefore, we can express the ex-post return r_t^b as a function of current and future inflation rates and future interest rates:

$$\hat{r}_t^b = \sum_{s=t+1}^{\infty} \frac{(1-\eta)^{s-t}}{(\bar{r}^b \bar{\pi})^{s-t-1}} \left(-\frac{\hat{\pi}_s}{\bar{\pi}} - \frac{\hat{r}_s^b}{\bar{r}^b \bar{\pi}} \right) - \frac{\bar{r}^b}{\bar{\pi}} \hat{\pi}_t.$$

By combining this equation with equation (31) and using the fact that $\bar{\pi} = 1$, we arrive at the fiscal financing decomposition condition (24):

$$\begin{aligned} \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{g}}{\bar{b}} \tilde{g}_s &= \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{T}}{\bar{b}} \tilde{T}_s - \frac{\hat{r}_t^b}{\bar{r}^b} - \sum_{s=t+1}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \hat{r}_s^b \\ &= \sum_{s=t}^{\infty} \frac{1}{(\bar{r}^b)^{s-t+1}} \frac{\bar{T}}{\bar{b}} \tilde{T}_s + \frac{1}{\bar{\pi}} \hat{\pi}_t + \sum_{s=t+1}^{\infty} \frac{(1-\eta)^{s-t}}{(\bar{r}^b)^{s-t}} \frac{\hat{\pi}_s}{\bar{\pi}} \\ &\quad - \sum_{s=t+1}^{\infty} \frac{1 - (1-\eta)^{s-t}}{(\bar{r}^b)^{s-t+1}} \hat{r}_s^b. \end{aligned}$$

A.7 Sensitivity of ϵ



$\epsilon = 1$.

Figure 9: Sensitivity analysis for ϵ .

Note: The vertical dashed line shows the benchmark value

Figure 9 reports the sensitivity of the fiscal financing decomposition to changes in the inverse Frisch elasticity ϵ . As ϵ varies from 0.2 to 4, the steady-state debt-to-GDP varies between 59.50% and 61.50%. The contribution of inflation in the fiscal financing varies between 45% and 60%, and it remains consistently and significantly smaller than its counterpart in the frictionless model.